

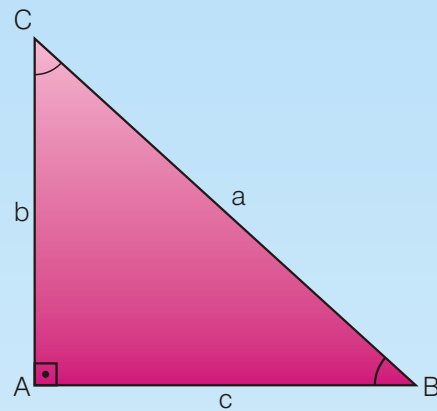


dicas do vestibular

Matemática: Trigonometria

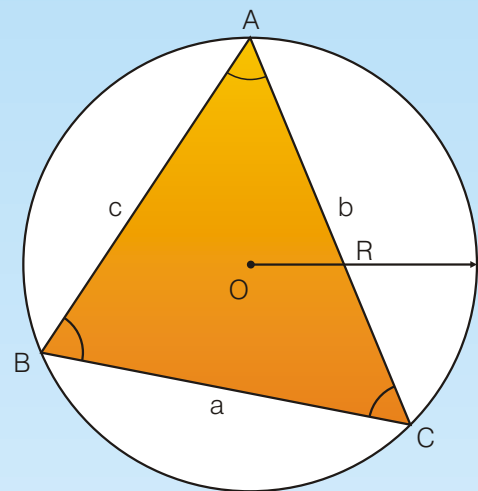
Material elaborado pelos professores Amauri e Tupy do Sistema de Ensino Energia.

1) Funções trigonométricas no triângulo retângulo



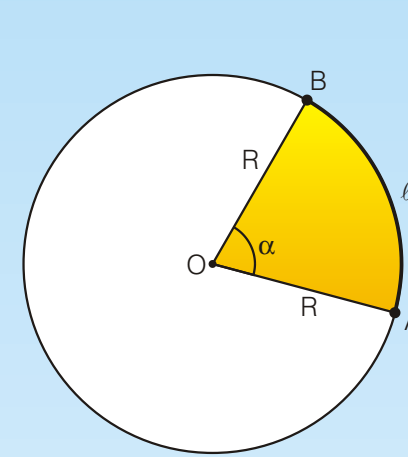
- $\text{sen } \hat{B} = \frac{\text{cateto oposto}}{\text{hipotenusa}} = \frac{b}{a}$
- $\text{cos } \hat{B} = \frac{\text{cateto adjacente}}{\text{hipotenusa}} = \frac{c}{a}$
- $\text{tg } \hat{B} = \frac{\text{cateto oposto}}{\text{cateto adjacente}} = \frac{b}{c}$

2) Relações num triângulo qualquer



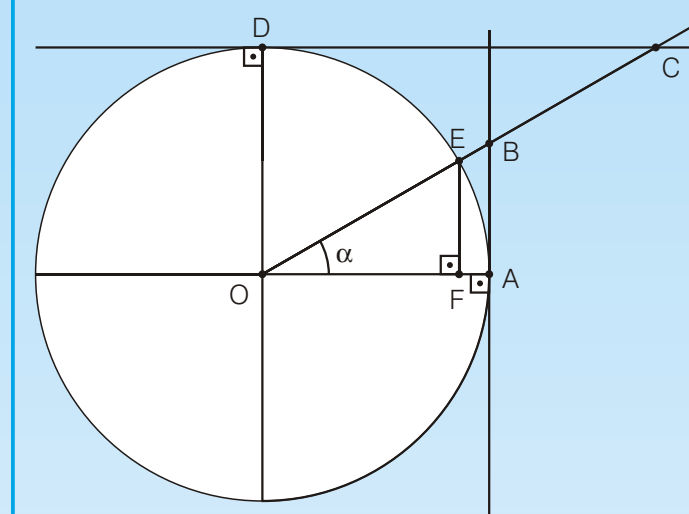
- Lei dos senos:
 $\frac{a}{\text{sen } \hat{A}} = \frac{b}{\text{sen } \hat{B}} = \frac{c}{\text{sen } \hat{C}} = 2R$
- Lei dos cossenos:
 $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \text{cos } \hat{A}$

3) Comprimento de um arco



- α : ângulo central (em radianos)
- $l = \alpha \cdot R$
- $180^\circ = \pi \text{ rad}$

4) Círculo e funções trigonométricas



- $\text{sen } \alpha = \overline{EF}$
- $\text{cos } \alpha = \overline{OF}$
- $\text{tg } \alpha = \overline{AB}$
- $\text{cotg } \alpha = \overline{DC}$
- $\text{sec } \alpha = \overline{OB}$
- $\text{cossec } \alpha = \overline{OC}$
- $\overline{OE} = R = 1$

5) Relações fundamentais e auxiliares

Fundamentais

- 1) $\text{sen}^2 x + \text{cos}^2 x = 1$
- 2) $\text{tg } x = \text{sen } x / \text{cos } x$
- 3) $\text{cotg } x = \text{cos } x / \text{sen } x$
- 4) $\text{sec } x = 1 / \text{cos } x$
- 5) $\text{cossec } x = 1 / \text{sen } x$

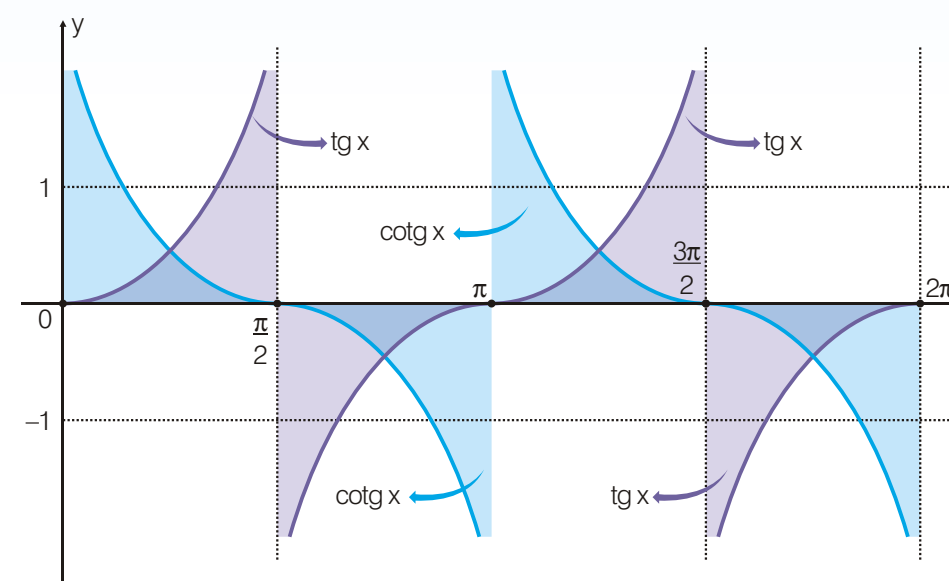
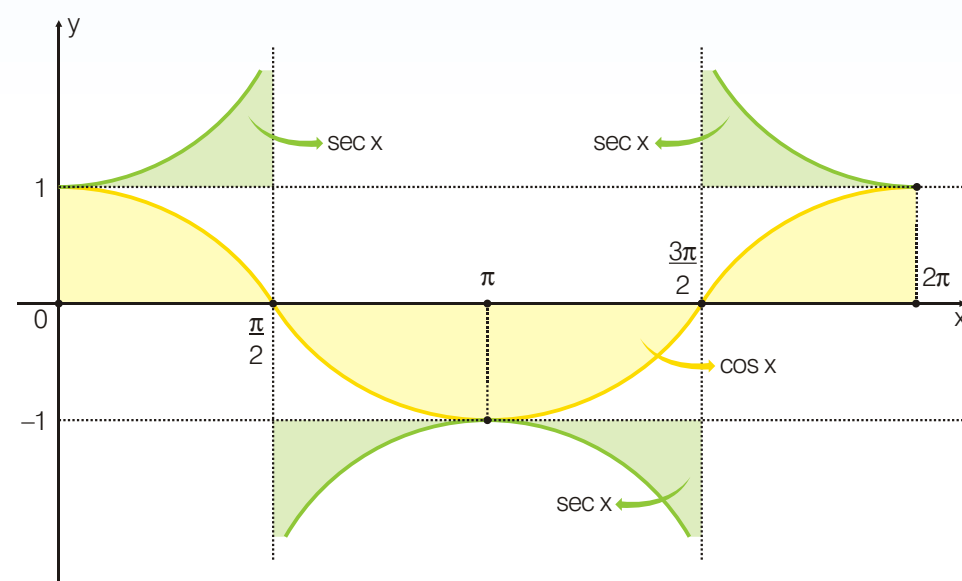
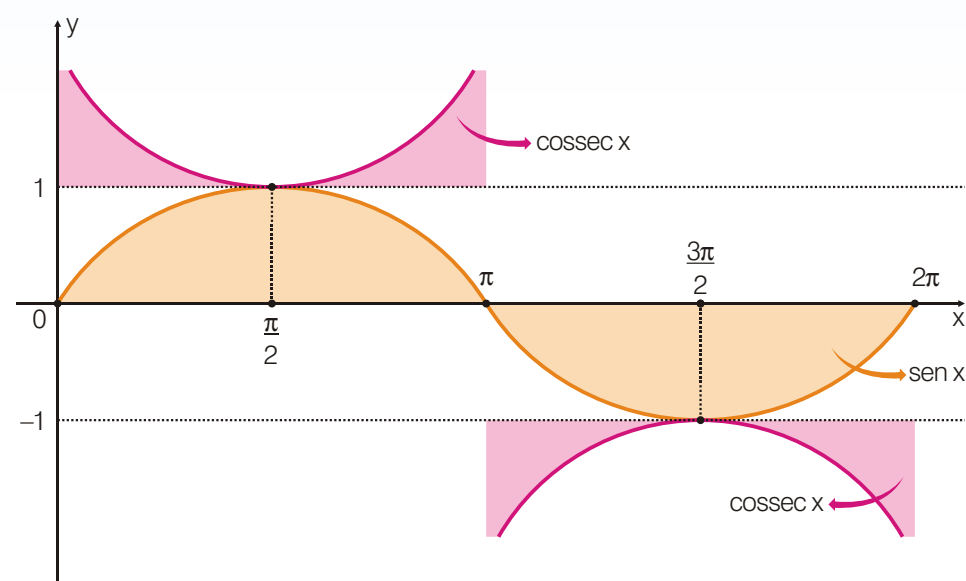
Auxiliares

- 1) $\text{sec}^2 x = 1 + \text{tg}^2 x$
- 2) $\text{cossec}^2 x = 1 + \text{cotg}^2 x$

6) Variações das funções básicas

Função	Domínio	Período	Imagem	Par ou ímpar	Sinais	Crescente ou decrescente			
						1º quadrante	2º quadrante	3º quadrante	4º quadrante
$y = a \pm b \cdot \text{sen } x$	\mathfrak{R}	2π	$[a - b, a + b]$	$\text{sen } x = -\text{sen } (-x) \rightarrow \text{ímpar}$	$\oplus \oplus$	crescente	decrescente	decrescente	crescente
$y = a \pm b \cdot \text{cos } x$	\mathfrak{R}	2π	$[a - b, a + b]$	$\text{cos } x = \text{cos } (-x) \rightarrow \text{par}$	$\oplus \oplus$	decrescente	decrescente	crescente	crescente
$y = a \pm b \cdot \text{tg } x$	$x \neq \frac{\pi}{2} + K\pi$	π	\mathfrak{R}	$\text{tg } x = -\text{tg } (-x) \rightarrow \text{ímpar}$	$\oplus \oplus$	crescente	crescente	crescente	crescente
$y = a \pm b \cdot \text{cotg } x$	$x \neq K\pi$	π	\mathfrak{R}	$\text{cotg } x = -\text{cotg } (-x) \rightarrow \text{ímpar}$	$\oplus \oplus$	decrescente	decrescente	decrescente	decrescente
$y = a \pm b \cdot \text{sec } x$	$x \neq \frac{\pi}{2} + K\pi$	2π	$(-\infty, a - b] \cup [a + b, +\infty)$	$\text{sec } x = \text{sec } (-x) \rightarrow \text{par}$	$\oplus \oplus$	crescente	crescente	decrescente	decrescente
$y = a \pm b \cdot \text{cossec } x$	$x \neq K\pi$	2π	$(-\infty, a - b] \cup [a + b, +\infty)$	$\text{cossec } x = -\text{cossec } (-x) \rightarrow \text{ímpar}$	$\oplus \oplus$	decrescente	crescente	crescente	decrescente

7) Gráficos das funções



8) Operações com arcos

Adição e subtração de arcos

$$\begin{aligned} \text{sen}(a \pm b) &= \text{sen } a \cdot \text{cos } b \pm \text{sen } b \cdot \text{cos } a \\ \text{cos}(a \pm b) &= \text{cos } a \cdot \text{cos } b \mp \text{sen } a \cdot \text{sen } b \\ \text{tg}(a \pm b) &= \frac{\text{tg } a \pm \text{tg } b}{1 \mp \text{tg } a \cdot \text{tg } b} \end{aligned}$$

Arco duplo

$$\begin{aligned} \text{sen } 2a &= 2 \cdot \text{sen } a \cdot \text{cos } a \\ \text{cos } 2a &= \text{cos}^2 a - \text{sen}^2 a \\ \text{tg } 2a &= \frac{2 \cdot \text{tg } a}{1 - \text{tg}^2 a} \end{aligned}$$

Arco triplo

$$\begin{aligned} \text{sen } 3a &= 3 \cdot \text{sen } a - 4 \cdot \text{sen}^3 a \\ \text{cos } 3a &= 4 \cdot \text{cos}^3 a - 3 \cdot \text{cos } a \end{aligned}$$

Bisseção de arcos

$$\begin{aligned} \text{sen } \frac{a}{2} &= \pm \sqrt{\frac{1 - \text{cos } a}{2}} \\ \text{cos } \frac{a}{2} &= \pm \sqrt{\frac{1 + \text{cos } a}{2}} \end{aligned}$$