

Matemática C – Semi-Extensivo – V. 4

Exercícios

01) Usando a regra de Chió, temos:

$$\begin{vmatrix} 3+2 & -1+2 & 5+2 \\ 9-4 & 1-4 & 25-4 \\ 27+8 & -1+8 & 125+8 \end{vmatrix} = \begin{vmatrix} 5 & 1 & 7 \\ 5 & 3 & 21 \\ 35 & 7 & 133 \end{vmatrix} \begin{vmatrix} 5 & 1 \\ 5 & 3 \\ 35 & 7 \end{vmatrix}$$

$$= 1995 + 735 + 245 - 735 - 735 - 665 = 840$$

02) Este é um determinante de Vandermonde.  
 $\det A = (\log 30 - \log 3) \cdot (\log 300 - \log 3) \cdot (\log 300 - \log 30) \cdot (\log 3000 - \log 3) \cdot (\log 3000 - \log 30) \cdot (\log 3000 - \log 300) \cdot (\log 10) \cdot \log 100 \cdot \log 10 \cdot \log 1000 \cdot \log 100 \cdot \log 10$   
 $\det A = 1 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1$   
 $\det A = 12$

03)  $\det(A) = 6 - 5$   
 $\det(A) = 1$

$$A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

04)  $\det A = 2 - 3$   
 $\det A = -1$

$$A^{-1} = \begin{bmatrix} 1 & -3 \\ -1 & -1 \\ -1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$$

$$\Rightarrow X = A^T - A^{-1}$$

$$X = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix}$$

05)  $\det A = 1$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow X = \left( \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \right)^T$$

$$X = \begin{pmatrix} -1 & 1+1 \\ 1 & -1 \end{pmatrix}^T \Rightarrow X = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$$

06) A

$$\frac{\det A^{-1} \cdot \det B \cdot \det(A)}{\det B} =$$

$$= \det(A^{-1}) \cdot \det A$$

$$= \frac{1}{\det A} \cdot \det A = 1$$

07) B

$$M \cdot A = 2B$$

$$M = 2B \cdot A^{-1}$$

$$\det M = \det(2B \cdot A^{-1})$$

$$\det M = 2^2 \cdot \det B \cdot \det A^{-1}$$

$$\det M = 4 \cdot \det B \cdot \frac{1}{\det A}$$

$$\det B = 8 - 7$$

$$\det B = 1$$

$$\det A = 6 - 4$$

$$\det A = 2$$

$$\Rightarrow \det M = 4 \cdot 1 \cdot \frac{1}{2}$$

$$\det M = 2$$

$$\det M^{-1} = \frac{1}{2}$$

08) C

$$\det A = 50 + 5x^2 + 4x - 2x^2 - 20 - 25x$$

$$\det A = 3x^2 - 21x + 30$$

$$3x^2 - 21x + 30 \neq 0 \div (7)$$

$$x^2 - 7x + 10 \neq 0$$

$$x' \neq 5$$

$$x'' \neq 2$$

09) 51

10) 02

11)  $\det(AB) = 8$

$$\det A \cdot \det B = 8$$

$$-1 \cdot \det B = 8$$

$$\det B = -8$$

$$\det A = -3 + 2$$

$$\det A = -1$$

12)  $\det(4A \cdot B^{-1}) =$

$$= \det(4A) \cdot \det B^{-1}$$

$$= 4^n \det A \cdot \frac{1}{\det B}$$

$$= \frac{4^n \cdot a}{b}$$

13)  $M^2 - 2M = 0$

$$\det(M^2 - 2M) = 0$$

$$\det M^2 - 2^2 \det M = 0$$

$$\det M \cdot \det M - 4 \det M = 0$$

$$\det^2 M - 4 \det M = 0$$

$$\det M \cdot (\det M - 4) = 0 \Rightarrow \det M = 0$$

$$\det M - 4 = 0$$

$$\det M = 4$$

14)  $\det(B^T) = \det B = 96$

$$\det A = 1,5$$

$$B = k \cdot A$$

$$\det B = \det(k \cdot A)$$

$$96 = k^3 \cdot \det A$$

$$96 = k^3 \cdot 1,5$$

$$\frac{96}{1,5} = k^3$$

$$k = \sqrt[3]{64}$$

$$k = 4$$

15)  $A \cdot B = C^{-1}$

$$\det(A \cdot B) = \det C^{-1}$$

$$\det A \cdot \det B = \frac{1}{32}$$

$$\det A \cdot \det(2A) = \frac{1}{32}$$

$$\det A \cdot 2^3 \cdot \det A = \frac{1}{32}$$

$$8\det^2 A = \frac{1}{32}$$

$$\det^2 A = \frac{1}{256}$$

$$\det A = \pm \frac{1}{16}$$

$$\Rightarrow |\det A| = \frac{1}{16}$$

16) 96

$$\det(A \cdot 2B) =$$

$$= \det A \cdot \det(2B)$$

$$= \det A \cdot 2^3 \cdot \det B$$

$$= 3 \cdot 8 \cdot 4$$

$$= 96$$

17) Obtendo-se a inversa através de determinantes, sabe-se que:

$$\det M = -1 + 6 + 6 + 4 - 9 - 1$$

$$\det M = 5$$

$$\Rightarrow x = \frac{1}{5} \cdot (-1)^{1+1} \cdot \begin{vmatrix} -1 & 3 \\ 3 & 1 \end{vmatrix}$$

$$x = \frac{1}{5} \cdot (-10)$$

$$x = -2$$

$$y = \frac{1}{5} \cdot (-1)^{2+2} \cdot \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$y = \frac{1}{5} \cdot (-3)$$

$$y = -\frac{3}{5}$$

$$z = \frac{1}{5} \cdot (-1)^{3+2} \cdot \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

$$z = -\frac{1}{5} \cdot 1$$

$$z = -\frac{1}{5}$$

$$w = \frac{1}{5} \cdot (-1)^{3+3} \cdot \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$w = \frac{1}{5} \cdot (-2)$$

$$w = -\frac{2}{5}$$

a)  $A = x - 3y - z + w$

$$A = -2 - 3 \cdot \left(-\frac{3}{5}\right) + \frac{1}{5} - \frac{2}{5}$$

$$A = -\frac{2}{5}$$

b)  $\det(2M) = 2^3 \cdot \det M$

$$\det(2M) = 8 \cdot 5$$

$$\det(2M) = 40$$

$$18) B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \Rightarrow B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1+2 & -1+1 \\ 1+6 & -2+3 \end{bmatrix} \Rightarrow 2AB = \begin{bmatrix} 6 & 0 \\ 14 & 2 \end{bmatrix}$$

$$\Rightarrow \det(2AB) = 12$$

19)  $c - 1 = 0 \Rightarrow c = 1$

$$b + c = 0 \Rightarrow b = -1$$

$$a - 3b = 0$$

$$a + 3 = 0 \Rightarrow a = -3$$

$$20) \begin{cases} x - 2 = 0 \Rightarrow x = 2 \\ y - 3 = 0 \Rightarrow y = 3 \end{cases} \begin{cases} ax - y = a \\ 4x - by = a + 2 \end{cases}$$

$$\begin{cases} 2a - 3 = a \Rightarrow a = 3 \\ 4 \cdot 2 - 3b = 5 \Rightarrow 3b = 3 \end{cases}$$

$$b = 1$$

21) Escalonamento

$$\left| \begin{array}{ccc|c} 2 & 1 & -4 & -1 \\ 4 & -2 & 8 & -2 \\ 8 & 1 & 4 & -10 \end{array} \right| \xrightarrow{l_1 \leftrightarrow l_2} \left| \begin{array}{ccc|c} 4 & -2 & 8 & -2 \\ 2 & 1 & -4 & -1 \\ 8 & 1 & 4 & -10 \end{array} \right| \rightarrow l_1 = \frac{l_1}{4}$$

$$\left| \begin{array}{ccc|c} 1 & -1/2 & 2 & -1/2 \\ 2 & 1 & -4 & -1 \\ 8 & 1 & 4 & -10 \end{array} \right| \xrightarrow{\begin{matrix} l_2 = l_2 - 2l_1 \\ l_3 = l_3 - 8l_1 \end{matrix}} \left| \begin{array}{ccc|c} 1 & -1/2 & 2 & -1/2 \\ 0 & 2 & -8 & 0 \\ 0 & 5 & -12 & -6 \end{array} \right| \rightarrow \frac{l_2}{2}$$

$$\left| \begin{array}{ccc|c} 1 & -1/2 & 2 & -1/2 \\ 0 & 1 & -4 & 0 \\ 0 & 5 & -12 & -6 \end{array} \right| \xrightarrow{l_3 = l_3 - 5l_2} \left| \begin{array}{ccc|c} 1 & -1/2 & 2 & -1/2 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 8 & -6 \end{array} \right|$$

$$\Rightarrow 8z = -6$$

$$z = -\frac{3}{4}$$

$$y - 4z = 0$$

$$y - 4 \cdot \left(-\frac{3}{4}\right) = 0$$

$$y = -3$$

$$x - \frac{y}{2} + 2z = -\frac{1}{2}$$

$$x + \frac{3}{2} - \frac{3}{2} = -\frac{1}{2}$$

$$x = -\frac{1}{2}$$

$$S = \left(-\frac{1}{2}, -3, -\frac{3}{4}\right)$$

22) Escalonamento

$$\left| \begin{array}{ccc|c} 2 & 3 & -1 & 3 \\ 3 & -2 & 1 & 2 \\ 1 & 1 & 0 & 1 \end{array} \right| \xrightarrow{l_1 \leftrightarrow l_3} \left| \begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 3 & -2 & 1 & 2 \\ 2 & 3 & -1 & 3 \end{array} \right| \begin{array}{l} l_2 = l_2 - 3l_1 \\ \longrightarrow \\ l_3 = l_3 - 2l_1 \end{array}$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -5 & 1 & 1 \\ 0 & 1 & -1 & 1 \end{array} \right| \xrightarrow{l_3 \leftrightarrow l_2} \left| \begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & -5 & 1 & 1 \end{array} \right| \longrightarrow \begin{array}{l} \\ \\ l_3 = l_3 + 5l_2 \end{array}$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 6 & 6 \end{array} \right| \rightarrow$$

$$6z = 6$$

$$z = 1$$

$$y - z = 1$$

$$y - 1 = 1$$

$$y = 2$$

$$x + y = 1$$

$$x + 2 = 1$$

$$x = -1$$

23) M: Matias; C: cachorro; G: gato

$$\begin{cases} M + C = 95 \\ M + G = 54 \\ C + G = 51 \end{cases}$$

$$\Rightarrow l_1 = l_2$$

$$M + C + M + G = 54 + 95$$

$$2M + C + G = 149$$

$$2M + 51 = 149$$

$$M = \frac{98}{2}$$

$$M = 49 \text{ kg}$$

$$M + G = 54$$

$$49 + G = 54$$

$$G = 5 \text{ kg}$$

$$C + G = 51$$

$$C + 5 = 51$$

$$C = 46 \text{ kg}$$

$$24) \begin{cases} 7x + y - 4z = 5a \\ x - 4y + az = b \end{cases} \text{ com } (x, y, z) = (2, -3, 1)$$

$$\begin{cases} 7 \cdot 2 - 3 - 4 \cdot 1 = 5a \\ 2 - 4 \cdot (-3) + a \cdot 1 = b \end{cases} \Rightarrow$$

$$\begin{cases} 14 - 7 = 5a \\ 2 + 12 + a = b \end{cases} \Rightarrow$$

$$\begin{cases} 7 = 5a \\ 14 + a = b \end{cases}$$

$$-14 - 3 - 4 = 5a$$

$$5a = 7$$

$$a = \frac{7}{5}$$

$$14 + a = b$$

$$14 + \frac{7}{5} = b$$

$$\frac{70+7}{5} = b$$

$$\frac{77}{5} = b$$

Agora, calculando  $a + b$  fica:

$$a + b = \frac{77}{5} + \frac{7}{5} = \frac{84}{5}$$

$$a + b = \frac{84}{5}$$

$$25) \begin{cases} 2x + 3y = 6 \\ 4x + 6y = 1 \end{cases}$$

Não é possível, pois:

$$2(2x + 3y) = 1$$

$$2 \cdot 6 = 1$$

$$12 = 1 \text{ (absurdo)}$$

26) C: coxinha; S: suco; Q: quibe

$$\begin{cases} C + S = 2 \\ C + Q = 2,20 \\ Q + S = 1,80 \end{cases}$$

$$\Rightarrow l_1 + l_2$$

$$C + S + C + Q = 2 + 2,20$$

$$2C + S + Q = 4,20$$

$$2C + 1,80 = 4,20$$

$$2C = 2,40$$

$$C = 1,20$$

27) P: pai; V: filho mais velho; N: filho mais novo

$$\begin{cases} V + N + 17 = P \\ 7 \cdot (N - 3) = P - 3 \\ 2 \cdot (V + 12) = P + 12 \end{cases}$$

$$7N - 21 = P - 3$$

$$N = \frac{P+18}{7}$$

$$2V + 24 = P + 12$$

$$V = \frac{P-12}{2}$$

$$V + N + 17 = P$$

$$\frac{P+18}{7} + \frac{P-12}{2} + 17 = P$$

$$\frac{2P + 36 + 7P - 84238}{14} = \frac{14P}{14}$$

$$190 = 14P - 9P$$

$$5P = 190$$

$$P = 38$$

$$28) \begin{cases} 2x - 0 + z = 4 \\ 5x + 0 - 3z = k \\ 0 + z = -2 \end{cases}$$

$$z = -2$$

$$2x - 2 = 4$$

$$x = 3$$

$$5x - 3z = k$$

$$5 \cdot 3 - 3 \cdot (-2) = k$$

$$k = 21$$

$$29) \begin{cases} x + y - 2z = -3 \\ x - 4y + az = b \\ 2x - y + z = 3 \end{cases}$$

Com  $-y + z = 1$ , substituímos em  $2x - y + z = 3$ :

$$2x - \underbrace{y + z}_1 = 3$$

$$2x + 1 = 3$$

$$2x = 2$$

$$x = 1$$

$$\begin{cases} x + y - 2z = -3 \\ 2x - y + z = 3 \end{cases} \Rightarrow$$

$$\begin{cases} 1 + y - 2z = -3 \\ 2 - y + z = 3 \end{cases} \Rightarrow$$

$$+ \begin{cases} y - 2z = -4 \\ -y + z = 1 \end{cases}$$

$$\hline -z = -3$$

$$z = 3$$

$$y - 2z = -4$$

$$y - 2 \cdot 3 = -4$$

$$y - 4 + 6$$

$$y = 2$$

Portanto,  $x = 1$ ,  $y = 2$  e  $z = 3$

$$30) (a - b)^2 = a^2 - 2ab + b^2$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$(a - b)^2 = 117 - 2 \cdot 54$$

$$(a - b)^2 = 117 - 108$$

$$(a - b)^2 = 9$$

$$31) \begin{cases} 2x + y = 4 \\ 3x + 3z - 4x + 4y = 10 \\ 4x - 3z + 3 = 10 \end{cases}$$

$$\begin{cases} 2x + y = 4 \\ -x + 4y + 3z = 10 \\ 4x - 3z = 7 \end{cases} \Rightarrow y = 4 - 2x$$

$$-x + 4y + 3z = 10$$

$$4x - 3z = 7$$

$$\begin{cases} -x + 4y + 3z = 10 \\ 4x - 3z = 7 \end{cases} \Rightarrow \begin{cases} -x + 4(4 - 2x) + 3z = 10 \\ 4x - 3z = 7 \end{cases}$$

$$\Rightarrow + \begin{cases} -9x + 3z = -6 \\ 4x - 3z = 7 \end{cases}$$

$$\hline 5x = 1$$

$$x = \frac{1}{5}$$

$$4x - 3z = 7$$

$$4 \cdot \frac{1}{5} - 3z = 7$$

$$-3z = \frac{31}{5}$$

$$z = -\frac{31}{15}$$

$$y = 4 - 2 \cdot \frac{1}{5}$$

$$y = \frac{18}{5}$$

$$2x - y - 3z$$

$$\frac{2}{5} - \frac{18}{5} + \frac{31}{5} = \frac{15}{5} = 3$$

$$32) \frac{1}{a} = x$$

$$\frac{1}{b} = y$$

$$\frac{1}{c} = z$$

$$\begin{cases} x + y + z = 2 \\ 2x + 3y + z = -1 \\ x - y - z = 0 \end{cases}$$

$$\ell_1 + \ell_3 \Rightarrow + \begin{cases} x + y + z = 2 \\ x - y - z = 0 \end{cases}$$

$$\hline 2x = 2$$

$$x = 1$$

$$\begin{cases} x + y + z = 2 \\ 2x + 3y + z = -1 \end{cases} \Rightarrow \begin{cases} 1 + y + z = 2 \\ 2 + 3y + z = -1 \end{cases}$$

$$\Rightarrow - \begin{cases} y + z = 1 \\ 3y + z = -3 \end{cases}$$

$$\hline -2y = 4$$

$$y = -2$$

$$y + z = 1$$

$$-2 + z = 1$$

$$z = 3$$

$$\frac{1}{a} = 1; a = 1$$

$$\frac{1}{b} = -2; b = -\frac{1}{2}$$

$$\frac{1}{c} = 3; c = \frac{1}{3}$$

$$33) \begin{cases} x - 3y = m \\ 2x + 3my = 4 \end{cases}$$

$$\begin{vmatrix} 1 & -3 \\ 2 & 3m \end{vmatrix}$$

$$3m + 6 \neq 0$$

$$3m \neq -6$$

$$m \neq -2$$

S.P.D.

$$(S.I.) m = -2$$

$$\begin{cases} x - 3y = -2 \\ 2x - 6y = 4 \end{cases} \Rightarrow \begin{vmatrix} -2 & -3 \\ 4 & -6 \end{vmatrix} = 12 + 12 = 24$$

34) C

$$\begin{vmatrix} 1 & 1 \\ 1 & a^2 \end{vmatrix}$$

$$a^2 - 1 = 0$$

$$a^2 = 1$$

$$a = \pm 1$$

$$\text{Se } a = -1 \Rightarrow \begin{cases} x + y = 1 \\ x + y = -1 \end{cases} \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \ell_2 = \ell_2 - \ell_1$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & -2 \end{vmatrix}$$

absurdo  $\rightarrow$  sistema impossível

$$35) \begin{vmatrix} 1 & 1 \\ 1 & m^2 \end{vmatrix}$$

$$m^2 - 1 \neq 0$$

$$m^2 \neq 1$$

$$m \neq \pm 1$$

$$\text{Se } m = 1 \Rightarrow \begin{cases} x + y = 1 \\ x + y = 1 \end{cases} \Rightarrow \text{S.P.I. } \ell_1 = \ell_2 \text{ (infinitas}$$

soluções)

$$\text{Se } m = -1 \Rightarrow \begin{cases} x + y = 1 \\ x + y = -1 \end{cases}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \ell_2 = \ell_2 - \ell_1 \rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & -2 \end{vmatrix}$$

$\hookrightarrow$  S.I.

(sem solução)

Logo,  $m \neq \pm 1$ .

$$36) \begin{vmatrix} 1 & 2 & 2 & 1 & 2 \\ 3 & 6 & -4 & 3 & 6 \\ 2 & b & -6 & 2 & b \end{vmatrix} = 0$$

$$-36 - 16 + 6b - 24 + 4b + 36 = 0$$

$$10b = 40$$

$$b = 4$$

$$\Rightarrow \begin{cases} x + 2y + 2z = a \\ 3x + 6y - 4z = 4 \\ 2x + 4y - 6z = 1 \end{cases}$$

$$\begin{vmatrix} 1 & 2 & 2 & a \\ 3 & 6 & -4 & 4 \\ 2 & 4 & -6 & 1 \end{vmatrix} \xrightarrow{\ell_2 = \ell_2 - \ell_1} \begin{vmatrix} 1 & 2 & 2 & a \\ 2 & 4 & -6 & 4 - a \\ 2 & 4 & -6 & 1 \end{vmatrix} \xrightarrow{\ell_3 = \ell_3 - \ell_2}$$

$$\begin{vmatrix} 1 & 2 & 2 & a \\ 2 & 4 & -6 & 4 - a \\ 0 & 0 & 0 & 1 - 4 + a \end{vmatrix} \rightarrow$$

Para ser S.P.I.:

$$1 - 4 + a = 0$$

$$a = 3$$

37) a)  $a = 1$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & -3 \end{vmatrix} \xrightarrow{\begin{matrix} \ell_2 = \ell_2 - \ell_1 \\ \ell_3 = \ell_3 - \ell_1 \end{matrix}} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -4 \end{vmatrix} \text{ S.I.}$$

b)  $a \neq 1$

38) D

$$a = b = 0 \Rightarrow \begin{cases} x + 3y - 4z = 0 \\ 3x + y = 0 \\ 4x = 0 \end{cases}$$

$$\Rightarrow \begin{vmatrix} 1 & 3 & -4 & 1 & 3 \\ 3 & 1 & 0 & 3 & 1 \\ 4 & 0 & 0 & 4 & 0 \end{vmatrix} = 16$$

S.P.D.: solução única, sistema homogêneo; solução trivial é única.

39) 11

40) Infinitas soluções – S.P.I. (sistema homogêneo)

$$\begin{vmatrix} m & -2 & -1 & m & -2 \\ 1 & -m & -2 & 1 & -m \\ 3 & -2 & 0 & 3 & -2 \end{vmatrix}$$

$$= 12 + 2 - 3m - 4m = 0$$

$$-7m = -14$$

$$m = 2$$

41) 21

$$42) a) \begin{vmatrix} 1 & 1 & m \\ 2 & 3 & -5 \\ 3 & -1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 2 & 3 \\ 3 & -1 \end{vmatrix} \neq 0$$

$$3 - 15 - 2m - 9m - 5 - 2 \neq 0$$

$$-11m \neq 19$$

$$m \neq -\frac{19}{11}$$

$$b) \begin{vmatrix} 1 & 1 & 0 & 3 \\ 2 & 3 & -5 & -7 \\ 3 & -1 & 1 & 4 \end{vmatrix} \begin{matrix} \xrightarrow{l_2=l_2-2l_1} \\ \xrightarrow{l_3=l_3-3l_1} \end{matrix} \begin{vmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & -5 & -13 \\ 0 & -4 & 1 & -5 \end{vmatrix} \begin{matrix} \\ \\ \xrightarrow{l_3=l_3+4l_2} \end{matrix}$$

$$\begin{vmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & -5 & -13 \\ 0 & 0 & -19 & -57 \end{vmatrix}$$

$$-19z = -57$$

$$z = 3$$

43) B

$$\begin{vmatrix} 1 & 1 & -1 & 3 \\ 1 & -1 & 1 & 1 \\ 1 & 3 & -3 & a \end{vmatrix} \begin{matrix} \xrightarrow{l_3=l_3-l_1} \\ \xrightarrow{l_2=l_2-l_1} \end{matrix} \begin{vmatrix} 1 & 1 & -1 & 3 \\ 0 & -2 & 2 & -2 \\ 0 & 2 & -2 & a-3 \end{vmatrix} \begin{matrix} \\ \\ \xrightarrow{l_3=l_3+l_2} \end{matrix}$$

$$\begin{vmatrix} 1 & 1 & -1 & 3 \\ 0 & -2 & 2 & -2 \\ 0 & 0 & 0 & a-5 \end{vmatrix}$$

$$a - 5 = 0$$

$$a = 5$$

$$44) \begin{cases} 4a - 3b + 20 = -9 \\ -8a + 6b - 40 = 18 \\ a - 3b + 20 = 6 \end{cases}$$

$$\begin{cases} 4a - 3b = -29 \\ a - 3b = -14 \end{cases}$$

$$3a = -15$$

$$a = -5$$

$$a - 3b = -14$$

$$-5 - 3b = -14$$

$$3b = 9$$

$$b = 3$$

$$a + 4b =$$

$$= -5 + 12 = 7$$

$$45) \begin{vmatrix} a+1 & 4 \\ 2 & a-1 \end{vmatrix}$$

$$(a+1) \cdot (a-1) - 8 = 0$$

$$a^2 - 1 - 8 = 0$$

$$a^2 = 9$$

$$a = \pm 3$$

$$46) \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & -1 \\ 1 & 3 & n \end{vmatrix} \begin{vmatrix} 1 & -1 \\ 3 & 1 \\ 1 & 3 \end{vmatrix}$$

$$n + 1 + 18 - 2 + 3 + 3n = 0$$

$$4n = -20$$

$$n = -5$$

$$\begin{vmatrix} 1 & -1 & 2 & 1 \\ 3 & 1 & -1 & m \\ 1 & 3 & -5 & 2m-5 \end{vmatrix} \begin{matrix} \\ \xrightarrow{l_3=l_3-l_1} \\ \xrightarrow{l_2=l_2-3l_1} \end{matrix}$$

$$\begin{vmatrix} 1 & -1 & 2 & 1 \\ 0 & 4 & -7 & m-3 \\ 0 & 4 & -7 & 2m-6 \end{vmatrix}$$

$$m - 3 = 2m - 6$$

$$m = 3$$

47) 06