Matemática E – Semi-Extensivo – V. 3

Exercícios

01) a) $\binom{4}{0} = \frac{4!}{0!4!} = 1$

b) $\binom{4}{1} = \frac{4!}{1!3!} = 4$

c) $\binom{4}{2} = \frac{4!}{2!2!} = 6$

d) $\binom{4}{3} = \frac{4!}{3!1!} = 4$

e) $\binom{4}{4} = \frac{4!}{4!0!} = 1$

02) a) $\binom{7}{4} = \frac{7!}{4!3!} = 35$

b) $\binom{12}{10} = \frac{12!}{10!2!} = 66$

c) $\binom{12}{2} = \frac{12!}{2!10!} = 66$

d) $\binom{8}{1} = \frac{8!}{1!7!} = 8$

e) $\binom{15}{1} = \frac{15!}{1!14!} = 15$

f) $\binom{n}{1} = \frac{n!}{1!(n-1)!} = n$

g) $\binom{7}{0} = \frac{7!}{0!7!} = 1$

h) $\binom{13}{0} = \frac{13!}{0!13!} = 1$

i) $\binom{n}{0} = \frac{n!}{0!n!} = 1$

03) A

$$\binom{200}{198} = \frac{200!}{198!2!} = \frac{\overset{100}{\cancel{200}} \,.\, 199 \,.\, \cancel{198!}}{\cancel{198!}\ \cancel{2!}} = 19900$$

04) $(2x + 1)^4 = \binom{4}{0} . (2x)^4 . 1^0 + \binom{4}{1} . (2x)^3 . 1^1 +$

$+ \binom{4}{2} . (2x)^2 . x^2 + \binom{4}{3} . (2x)^1 \ . 1^3 + \binom{4}{4} . (2x)^0 . 1^4 =$

$= 1 . 16x^4 + 4 . 8x^3 + 6 . 4x^2 + 4 . 2x + 1 =$

$= 16x^4 + 32x^3 + 24x^2 + 8x + 1$

Soma dos coeficientes: $16 + 32 + 24 + 8 + 1 = 81$

05) $\left(2x + \frac{1}{x}\right)^5$

$T_3 = \binom{5}{2} . (2x)^3 . \left(\frac{1}{x}\right)^2 =$

$= 10 . 8x^3 . \frac{1}{x^2} =$

$= 80x$

Coeficiente: 80

06) a) $(x + 2)^4 = \binom{4}{0} x^4 . 2^0 + \binom{4}{1} . x^3 . 2^1 +$

$+ \binom{4}{2} . x^2 . 2^2 + \binom{4}{3} x^1 . 2^3 + \binom{4}{4} x^0 . 2^4 =$

$= 1 . x^4 + 4 . x^3 . 2 + 6 . x^2 . 4 + 4 . x . 8 + 1 . 16 =$

$= x^4 + 8x^3 + 24x^2 + 32x + 16$

b) $(x - 2)^4 = \binom{4}{0} x^4 . (-2)^0 + \binom{4}{1} x^3 . (-2)^1 +$

$+ \binom{4}{2} x^2 . (-2)^2 + \binom{4}{3} x^1 . (-2)^3 + \binom{4}{4} x^0 . (-2)^4 = 1 . x^4$

$- 4 . x^3 . 2 + 6 . x^2 . 4 - 4 . x . 8 + 1 . 16 =$

$= x^4 - 8x^3 + 24x^2 - 32x + 16$

c) $(2x + y)^3 = \binom{3}{0} (2x)^3 . y^0 + \binom{3}{1} (2x)^2 . y^1 +$

$+ \binom{3}{2} (2x)^1 . y^2 + \binom{3}{3} (2x)^0 . y^3 =$

$= 1 . 8x^3 + 3 . 4x^2 . y + 3 . 2x . y^2 + 1. y^3 =$

$= 8x^3 + 12x^2y + 6xy^2 + y^3$

07) $\left(x^3 + \frac{2}{x}\right)^6$; $T_{p+1} = \binom{n}{p} x^{n-p} . a^p$

a) $T_1 = \binom{6}{0} . (x^3)^{6-0} . \left(\frac{2}{x}\right)^0 =$

$= 1 . x^{18} . 1 =$

$= x^{18}$

b) $T_4 = \binom{6}{3} . (x^3)^{6-3} . \left(\frac{2}{x}\right)^3 =$

$= 20 . x^9 . \frac{8}{x^3} =$

$= 160x^6$

c) $T_7 = \binom{6}{6} . (x^3)^{6-6} . \left(\frac{2}{x}\right)^6 =$

$= 1 . 1 . \frac{64}{x^6} =$

$= \frac{64}{x^6} = 64 . x^{-6}$

08) D

$\left(2x^2 - \frac{\sqrt{2}}{2}\right)^m$

7 termos

$m + 1 = 7$

$m = 6$

$T_3 = \binom{6}{2} . (2x^2)^4 . \left(-\frac{\sqrt{2}}{2}\right)^2 =$

$= 15 . \cancel{16}x^8 . \frac{2}{\cancel{4}} =$

$= 120x^8$

09) D

$(x^2 - 2)^5 = x^{10} + mx^8 + 40x^6 - 80x^4 + 80x^2 + n$

2º termo

$T_2 = \binom{5}{1} (x^2)^4 . (-2)^1 =$

$= 5x^8 . (-2) =$

$= -10x^8$

$m = -10$

6º termo:

$T_6 = \binom{5}{5} . (x^2)^0 . (-2)^5 =$

$= 1 . (-32) =$

$= -32$

$n = -32$

Logo, $m + n = -10 - 32 = -42$.

10) D

$\left(x^3 + \frac{2}{x}\right)^3$

Soma dos coeficientes: $(1 + 2)^3 = 3^3 = 27$

11) A

$(2x + 3y)^6$

Soma dos coeficientes: $(2 + 3)^6 = 5^6 = 15625$

12) B

$(2x + 3y)^8$

Termo médio $= T_5$

$T_5 = \binom{8}{4} . (2x)^4 . (3y)^4 = 70 . 16x^4 . 81y^4$

13) $\left(\frac{2}{x} - x\right)^{10}$

Termo central: T_6

$T_6 = \binom{10}{5} . \left(\frac{2}{x}\right)^5 . (-x)^5 = -252 . \frac{32}{\cancel{x^5}} . \cancel{x^5} = -8064$

14) a) $(x + 2)^6$

$T_3 = \binom{6}{2} x^4 . 2^2 = 15x^4 . 4 = 60x^4$

b) $(x + 2)^6$

Termo médio $= T_4$

$T_4 = \binom{6}{3} x^3 . 2^3 = 20x^3 . 8 = 160x^3$

c) $(x + 2)^6$

$T_{p+1} = \binom{6}{p} x^{6-p} . 2^p$

$6 - p = 5 \Rightarrow p = 1$

$T_2 = \binom{6}{1} x^5 . 2^1 = 6x^5 . 2 = 12x^5$

Coeficiente: 12

d) $(x + 2)^6$

$T_{p+1} = \binom{6}{p} x^{6-p} . 2^p$

$6 - p = 0 \Rightarrow p = 6$

$T_7 = \binom{6}{6} x^{6-6} . 2^6 = 1 . x^0 . 64 = 64$

15) B

$P(x) = (x + 2)^6$

$T_{p+1} = \binom{6}{p} x^{6-p} . 2^p$

$6 - p = 4$

$p = 2$

$T_3 = \binom{6}{2} x^{6-2} \,.\, 2^2 = 15 \,.\, x^4 \,.\, 4 = 60x^4$

Coeficiente: 60

16) D

$(3 + 2x)^5$

$T_{p+1} = \binom{5}{p} 3^{5-p} \,.\, (2x)^p =$

$= \binom{5}{p} 3^{5-p}\, 2^p \,.\, x^p$

$p = 3$

$T_4 = \binom{5}{3} 3^{5-3} \,.\, 2^3 \,.\, x^3 = 10 \,.\, 9 \,.\, 8x^3 = 720x^3$

Coeficiente: 720

17) $\left(\sqrt{x} + \frac{1}{x}\right)^{15}$

$T_{p+1} = \binom{n}{p} \,.\, x^{n-p} \,.\, a^p =$

$= \binom{15}{p} \,.\, \left(\sqrt{x}\right)^{15-p} . \left(\frac{1}{x}\right)^p =$

$= \binom{15}{p} \,.\, x^{\frac{15-p}{2} - p} \,.\, x^{-p} =$

$= \binom{15}{p} \,.\, x^{\frac{15-p}{2} - p}$

$\frac{15-p}{2} - p = 0$

$\frac{15-p}{2} = p$

$15 - p = 2p$

$p = 5$

$T_6 = \left(\frac{15}{5}\right) \,.\, x^0 =$

$= \frac{15!}{5!10!} = \frac{\overset{3}{\cancel{15}} . \overset{7}{\cancel{14}} . 13 . \cancel{12} . 11 . \cancel{10}!}{\cancel{5} . \cancel{4} . \cancel{3} . \cancel{2} . 1 . \cancel{10}!} =$

$= 3003$

18) B

$\left(x^4 - \frac{1}{x}\right)^{10}$

$T_{p+1} = \binom{10}{p} (x^4)^{10-p} \,.\, \left(\frac{-1}{x}\right)^p =$

$= \binom{10}{p} x^{40-4p} \,.\, x^{-p} \,.\, (-1)^p =$

$= \binom{10}{p} x^{40-5p} \,.\, (-1)^p$

$40 - 5p = 0$

$p = 8$

$T_9 = \binom{10}{8} x^0 \,.\, (-1)^8 = 45$

19) $\left(-x + \frac{\sqrt{2}}{x}\right)^8$

$T_{p+1} = \binom{8}{p} (-x)^{8-p} \,.\, \left(\frac{\sqrt{2}}{x}\right)^p =$

$= \binom{8}{p} (-1)^{8-p} \,.\, x^{8-p} \,.\, x^{-p} \,.\, (\sqrt{2})^p =$

$= \binom{8}{p} (-1)^{8-p} \,.\, x^{8-2p} \,.\, (\sqrt{2})^p$

$8 - 2p = 0$

$p = 4$

$T_5 = \binom{8}{4} (-1)^{8-4} \,.\, x^0 \,.\, (\cancel{\sqrt{2}})^{\cancel{4}} = 70 \,.\, 2^2 = 280$

20) $\left(xy - \frac{1}{y^2}\right)^6$

$T_{p+1} = \binom{6}{p} (xy)^{6-p} \,.\, \left(\frac{-1}{y^2}\right)^p =$

$= \binom{6}{p} x^{6-p} \,.\, y^{6-p} \,.\, (-1)^p \,.\, y^{-2p} =$

$= \binom{6}{p} x^{6-p} \,.\, y^{6-3p} \,.\, (-1)^p$

$6 - 3p = 0$

$p = 2$

$T_3 = \binom{6}{2} x^{6-2} \,.\, y^0 \,.\, (-1)^2 = 15 \,.\, x^4$

Coeficiente: 15

21) $T = \binom{6}{p} \,.\, x^p \,.\, \left(\frac{-1}{x^2}\right)^{6-p}$

$T = \binom{6}{p} \,.\, (-1)^{6-p} \,.\, x^p \,.\, x^{-2\,.\,(6-p)}$

$T = \binom{6}{p} \,.\, (-1)^{6-p} \,.\, x^{3p-12}$

$3p - 12 = 0$

$3p = 12$

$p = 4$

$T = \binom{6}{4} \,.\, (-1)^{6-4} \,.\, x^{3\,.\,4-12}$

$T = \dfrac{6!}{4!2!} \,.\, 1 \,.\, 1$

$T = \dfrac{720}{24\,.\,2}$

$T = \dfrac{720}{48}$

$T = 15$

22) $P(x) = 4x^3 + 5x^2 - 7x - 2$
a) $P(1) = 4 + 5 - 7 - 2 = 0$
b) $P(-2) = -32 + 20 + 14 - 2 = 0$
c) $P(0) = 0 + 0 - 0 - 2 = -2$

23) $P(x) = mx^2 - 5x + 2$
$P(-2) = 5$
$4m + 10 + 2 = 5$
$4m = -7$
$m = -\dfrac{7}{4}$

24) $P(x) = 2x^3 + px^2 - 5x + q$
$P(1) = 7 \Rightarrow 2 + p - 5 + q = 7$
$P(2) = 25 \Rightarrow 16 + 4p - 10 + q = 25$

$$\begin{cases} p + q = 10 & .(-1) \\ 4p + q = 19 \end{cases}$$

$$\begin{cases} -p - \cancel{q} = -10 \\ 4p + \cancel{q} = 19 \end{cases} \oplus$$

$3p = 9$
$p = 3;\ q = 7$

25) $P(x) = x^3 + ax^2 - 2x - 1$
a) $P(3) = 2$
$27 + 9a - 6 - 1 = 2$
$9a = -18$
$a = -2$
b) 1 é raiz. $\Rightarrow P(1) = 0$
$1 + a - 2 - 1 = 0$
$a = 2$

26) $P(x) = ax + b$
$P(0) = 3 \Rightarrow b = 3$
$P(-1) = 2 \Rightarrow -a + b = 2 \Rightarrow a = 1$
$P(x) = x + 3$

27) $P(x) + x \,.\, P(2 - x) = x^2 + 3$
$x = 0$
$P(0) + 0 \,.\, P(2 - 0) = 0 + 3$
$P(0) = 3$

$x = 1$
$P(1) + 1 \,.\, P(2 - 1) = 1 + 3$
$2P(1) = 4$
$P(1) = 2$

$x = 2$
$P(2) + 2 \,.\, P(2 - 2) = 4 + 3$
$P(2) + 2 \,.\, P(0) = 7$
$P(2) + 6 = 7$
$P(2) = 1$

28) E
$P(x) = x^3 + ax^2 + bx + c$
$P(1) = 0 \Rightarrow 1 + a + b + c = 0$
$a + b + c = -1$
$P(-x) + P(x) = 0$
$-\cancel{x^3} + ax^2 - \cancel{bx} + c + \cancel{x^3} + ax^2 + \cancel{bx} + c = 0$
$2ax^2 + 2c = 0$

$$\begin{cases} 2a = 0 \Rightarrow a = 0 \\ 2c = 0 \Rightarrow c = 0 \end{cases}$$

Como $a + b + c = -1$:
$b = -1$
Assim, $P(x) = x^3 - x$.
$P(2) = 8 - 2 = 6$

29) C
P(x) tem grau 5.
Exemplo: $P(x) = x^5$
$[P(x)]^3 + [P(x)]^2 + 2P(x) =$
$= (x^5)^3 + (x^5)^2 + 2 \,.\, x^5 =$
$= x^{15} + x^{10} + 2x^5$
Grau 15
Observação: O grau é 15 independentemente do polinômio P(x) tomado como exemplo.

30) $P(x) = (a - 3)x^3 + (b + 2a)x^2 + (6b + c)x$
$P(x) \equiv 0$
$a - 3 = 0$
$a = 3$
$b + 2a = 0$
$b + 6 = 0$
$b = -6$
$6b + c = 0$
$-36 + c = 0$
$c = 36$
$2 \,.\, (a + b + c)$
$2 \,.\, (3 - 6 + 36) = 66$

31) $P_1(x) = a(x + c)^3 + b(x + d) =$
$= a \,.\, (x^3 + 3x^2c + 3xc^2 + c^3) + bx + bd =$
$= ax^3 + 3acx^2 + 3ac^2x + ac^3 + bx + bd =$
$= ax^3 + 3acx^2 + (3ac^2 + b)x + ac^3 + bd$
$P_2(x) = x^3 + 6x^2 + 15x + 14$
$P_1(x) \equiv P_2(x)$
$a = 1$
$3ac = 6$
$c = 2$
$3ac^2 + b = 15$
$12 + b = 15$
$b = 3$
$ac^3 + bd = 14$
$8 + 3d = 14$
$3d = 6$
$d = 2$

32) a) **Falso.**

$k = 4 \Rightarrow P(x) = (4 - 4)x^2 + 4^2x + 2$

$P(x) = 16x + 2$

Grau = 1

b) **Falso.**

É necessário que $k - 4 \neq 0 \Rightarrow k \neq 4$.

c) **Verdadeiro.**

$P(x) = (k - 4) \,.\, x^2 + k^2x + 2 = 0$

Para $x = -k$

$P(-k) = (k - 4) \,.\, K^2 + k^2 \,.\, (-K) + 2 = 0$

$P(-k) = K^3 - 4k^2 - k^3 + 2$

$P(-K) = -4k^2 + 2$

d) **Falso.**

-1 é raiz $\Rightarrow P(-1) = 0$

$(k - 4) \,.\, (-1)^2 + k^2 \,.\, (-1) + 2 = 0$

$k - 4 - k^2 + 2 = 0$

$-k^2 + k - 2 = 0 \qquad (-1)$

$k^2 - k + 2 = 0$

$\Delta = 1 - 4 \,.\, 1 \,.\, 2 = -7$

$S = \varnothing$

e) **Verdadeiro.**

$k = 5 \Rightarrow P(x) = (5 - 4) \,.\, x^2 + 5^2 .\, x + 2$

$X^2 + 25X + 2 = 0$

$\Delta = (25)^2 - 4 \,.\, 1 \,.\, 2$

$\Delta = 617 > 0$

P(x) admite raízes reais.

33) $P(x) = ax^5 + (b - a + 1) \,.\, x^3 + (c + b + a) \,.\, x \equiv 0$

$a = 0$

$b - a + 1 = 0 \Rightarrow b = -1$

$c + b + a = 0 \Rightarrow c = 1$

34) $P(x) = ax^4 + bx^3 + c$

$Q(x) = ax^3 - bx + c$

$P(0) = 0 \Rightarrow c = 0$

$$\begin{matrix} P(1) = 0 \Rightarrow \\ Q(1) = 6 \Rightarrow \end{matrix} \begin{cases} a + b = 0 \\ a - b = 6 \end{cases} \oplus$$

$2a = 6$

$a = 3$

$b = -3$

$c = 0$

35) $P(x) = ax^2 + bx + c$

$P(1) = 4 \Rightarrow a + b + c = 4$

$P(0) + P(-1) = 3 \Rightarrow c + a - b + c = 3$

$P(-2) = -5 \Rightarrow 4a - 2b + c = -5$

$$\begin{cases} a + b + c = 4 \Rightarrow c = 4 - a - b & \text{(I)} \\ a - b + 2c = 3 & \text{(II)} \\ 4a - 2b + c = -5 & \text{(III)} \end{cases}$$

Substituindo I em II e III, temos:

$$\begin{cases} a - 2 + 2 \,.\, (4 - a - b) = 3 \\ 4a - 2b + 4 - a - b = -5 \end{cases}$$

$$\begin{cases} -a - 2b = -3 \\ 3a - 3b = -9 \quad \div (3) \end{cases}$$

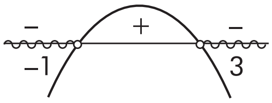
$$\begin{cases} -\cancel{a} - 2b = -3 \\ \cancel{a} - b = -3 \end{cases} \oplus$$

$-3b = -6$

$b = 2;\ a = -1;\ c = 3$

$P(x) = -x^2 + 2x + 3;\ P(x) < 0$

$x' = -1;\ x'' = 3$



$P(x) < 0$ para $x < -1$ ou $x > 3$

36) $P_1(x) = (m + n + p) \,.\, x^4 - (p + 1) \,.\, x^3 + mx^2 +$
$+ (n - p) \,.\, x + n$

$P_2(x) = 2mx^3 + (2p + 7) \,.\, x^2 + 5mx + 2m$

$P_1(x) \equiv P_2(x)$

$n = 2m$

$n - p = 5m \Rightarrow 2m - p = 5m \Rightarrow p = -3m$

$m = 2p + 7 \Rightarrow m = -6m + 7 \Rightarrow m = 1$

$p = -3$

$n = 2$

$m + n + p = 0$

Isto ocorre, pois $m = 1$; $p = -3$ e $n = 2$.

37) C

$P(x) = x^2 + 2x - 8$

$Q(x) = m \,.\, (x - 2) \,.\, (x - 3) + n \,.\, (x - 1) \,.\, (x - 2) =$
$= m \,.\, (x^2 - 3x - 2x + 6) + n \,.\, (x^2 - 2x - x + 2) =$
$= mx^2 - 5mx + 6m + nx^2 - 3nx + 2n =$
$= (m + n) \,.\, x^2 + (-5m - 3n) \,.\, x + 6m + 2n$

$P(x) \equiv Q(x)$

$$\begin{cases} m + n = 1 & \cdot (3) \\ -5m - 3n = 2 \end{cases}$$

$$\begin{cases} 3m + \cancel{3n} = 3 \\ -5m - \cancel{3n} = 2 \end{cases} \oplus$$

$-2m = 5$

$m = -\dfrac{5}{2}$

$n = \dfrac{7}{2}$

Observe que a terceira igualdade fica satisfeita $6m + 2n = -8$ para os valores obtidos de **m** e **n**.

38) $P_2(x) = 10x^2 + 158x + 29$

$P_1(x) = ax^2 + (b + c) \,.\, x - 2a - 3x^2 + 3cx + 3b + 1 =$
$= (a - 3) \,.\, x^2 + (b + 4c) \,.\, x - 2a + 3b + 1$

$P_1(x) \equiv P_2(x)$

$a - 3 = 10 \Rightarrow a = 13$

$-2a + 3b + 1 = 29 \Rightarrow -26 + 3b + 1 = 29$

$b = 18$

$b + 4c = 158 \Rightarrow 18 + 4c = 158 \Rightarrow c = 35$

Logo, $a + b + c = 13 + 18 + 35 = 66$.

39) D

$P(x) = px^2 + qx - 4$

$Q(x) = x^2 + px + q$

$P(x + 1) = p(x + 1)^2 + q(x + 1) - 4 =$
$= p(x^2 + 2x + 1) + qx + q - 4 =$
$= px^2 + (2p + q) . x + p + q - 4 =$
$Q(2x) = (2x^2) + p . (2x) + q =$
$= 4x^2 + 2px + q$
Como $P(x + 1) \equiv Q(2x)$:
$p = 4$
$2p + q = 2p \Rightarrow q = 0$
$p + \cancel{q} - 4 = \cancel{q} \Rightarrow p = 4$

40) C

$$\frac{2x-4}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$\frac{2x-4}{x^2-1} = \frac{Ax - A + Bx + B}{(x+1) . (x-1)}$$

$$\frac{2x-4}{\cancel{x^2-1}} = \frac{(A+B) . x - A + B}{\cancel{x^2-1}}$$

$2x - 4 = (A + B) . x - A + B$

$$\begin{cases} \cancel{A} + B = 2 \\ -\cancel{A} + B = -4 \end{cases} \oplus$$

$2B = -2$
$B = -1; A = 3$

41) E

$$\frac{8}{x^3 - 4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$\frac{8}{x^3 - 4x} \pm \frac{A(x^2 - 4) + B(x^2 + 2x) + C(x^2 - 2x)}{x . (x-2) . (x+2)}$$

$$\frac{8}{\cancel{x^3 - 4x}} = \frac{Ax^2 - 4A + Bx^2 + 2Bx + Cx^2 - 2Cx}{\cancel{x . \left(x^2 - 4\right)}}$$

$8 = (A + B + C) . x^2 + (2B - 2C) . x - 4A$
$-4A = 8 \Rightarrow A = -2$
$A + B + C = 0 \Rightarrow B + C = 2$ (I)
$2B - 2C = 0 \div (2) \Rightarrow B - C = 0$ (II)
De I e II, obtemos:

$$\begin{cases} B + \cancel{C} = 2 \\ B - \cancel{C} = 0 \end{cases} \oplus$$

$2B = 2$
$B = 1; C = 1$

42) B
$P(x) = 1 + 2x + 3x^2 + ... + 49x^{48} + 50x^{49}$

I. **Correta.**
$P(1) = 1 + 2 + 3 + ... + 49 + 50$

$P(1) = \dfrac{(50 + 1) . 50}{2}$ (soma de P.A.)

$P(1) = 1275$

II. **Incorreta.**
$P(-1) = 1 - 2 + 3 - 4 + ... - 48 + 49 - 50$
Há duas P.A..
$1 + 3 + 5 + ... + 47 + 49$ (25 termos)
$-2 - 4 - 6 ... - 48 - 50$ (25 termos)

$$S_{25} = \frac{(1 + 49) . 25}{2}$$

$S_{25} = 625$

$$S_{25} = \frac{(-2 - 50) . 25}{2}$$

$S_{25} = -650$
$P(-1) = 625 - 650$
$P(-1) = -25$

III. **Incorreta.**
$P(0) = 1 + 2 . 0 + 3 . 0 + 4 . 0 + ... + 49 . 0 +$
$+ 50 . 0$
$P(0) = 1$

IV. **Correta.**
$2x + 4x^3 + 6x^5 + 8x^6 + ... + 48x^{47} + 50x^{49}$
25 termos

$$S_n = \frac{(2 + 50) . 25}{2}$$

$S_n = 650$

43) $\dfrac{x^2 + x + 2}{x^3 - x} = \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{x-1}$

$$\frac{x^2 + x + 2}{x(x-1) . (x+1)} = \frac{A(x+1) . (x-1) + Bx(x-1) + Cx(x+1)}{x(x-1) . (x+1)}$$

$$\frac{x^2 + x + 2}{x(x-1) . (x+1)} = \frac{Ax^2 - A + Bx^2 - Bx + Cx^2 + Cx}{x(x-1) . (x+1)}$$

$$\frac{x^2 + x + 2}{x(x-1) . (x+1)} = \frac{x^2(A + B + C) + x(C - B) - A}{x(x-1) . (x+1)}$$

$-A = 2$
$A = -2$

$$+\begin{cases} A + B + C = 1 \\ -B + C = 1 \end{cases}$$

$A + 2C = 2$
$-2 + 2C = 2$
$C = 2$
$-B + C = 1$
$-B + 2 = 1$
$B = 1$
a) $A^2 + 2B - C^3 =$
$= (-2)^2 + 2 . 1 - (2^3) = -2$
b) $x = 0$
$x + 1 \neq 0 \Rightarrow x \neq -1$
$x - 1 \neq 0 \Rightarrow x \neq 1$

44) $P(x) = ax^2 + bx + c$
$P(1) - P(0) = 1 \Rightarrow$
$a . 1^2 + b . 1 + c - a . 0^2 - b . 0 - c = 1$
$a + b = 1$
$P(1) + P(0) = 3 \Rightarrow$
$a . 1^2 + b . 1 + c + a . 0^2 + b . 0 c = 3$
$a + b + 2c = 3$
$P(-1) - P(1) = -4 \Rightarrow$
$a . (-1)^2 + b . (-1) + c - a . 1^2 - b . 1 - c = -4$

$-2b = -4$
$b = 2$
$a + b = 1$
$a + 2 = 1$
$a = -1$
$a + b + 2c = 3$
$-1 + 2 + c = 3$
$c = 1$
$P(x) = -x^2 + 2x + 1$

45) E
$P_1(x) = 2x^2 - 3x + 1$
$P_2(x) = 3x - 1$
$P_3(x) = 6x^3 - 11x^2 + 6x - 1$
Dividindo $P_3(x)$ por $P_1(x)$, temos:
$(6x^3 - 11x^2 + 6x - 1) =$
$= (2x^2 - 3x + 1) \,.\, (ax + b) + cx + d =$
$= 2ax^3 + 2bx^2 - 3ax^2 - 3bx + ax + b + cx + d =$
$= 2ax^3 + (2b - 3a)x^2 + (a - 3b + c)x + b + d$
$2a = 6 \Rightarrow a = 3$
$2b - 3a = -11 \Rightarrow b = -1$
$a - 3b + c = 6 \Rightarrow c = 0$
$b + d = -1$
$d = 0$
Logo, $R(x) = 0$, e $P_3(x)$ é divisível por $P_1(x)$.

46) a) **Verdadeira**.
b) **Verdadeira**.
c) **Falsa**.
Se, por exemplo, $n = 3$ e
$P(x) = x^3 + 2$ e
$Q(x) = -x^3 + 7$, então
$P(x) + Q(x) = 9$ não tem grau 3.
d) **Verdadeira**.
e) **Falsa**.
Se $gr(P) = 5$ e $gr(Q) = 3$, então $gr(P + Q) = 5$.
f) **Falsa**.
Se $P(x) = x^5$ e $Q(x) = x^3$, então $P(x) \,.\, Q(x) = x^8$.

47) D
$P(x) \equiv D(x) \,.\, Q(x) + R(x)$
$ax^3 + bx^2 + cx + d = (x^2 + 1) \,.\, (2x - 1) +$
$+ (-x + 1) =$
$= 2x^3 - x^2 + 2x - 1 - x + 1 =$
$= 2x^3 - x^2 + x$
$a = 2;\ b = -1;\ c = 1;\ d = 0$
$P(x) = 2x^3 - x^2 + x$

48) $x^3 + px^2 + qx - 1 = (x^2 - x + 2) \,.\, (ax + b) - 7 =$
$= ax^3 + bx^2 - ax^2 - bx + 2ax + 2b - 7 =$
$= ax^3 + (b - a) \,.\, x^2 + (2a - b) \,.\, x + 2b - 7$
$a = 1$
$2b - 7 = -1 \Rightarrow b = 3$
$b - a = p \Rightarrow p = 2$
$2a - b = q \Rightarrow q = -1$

49) E
Divisível $\Rightarrow R(x) = 0$
$2x^4 - 3x^3 + ax^2 + bx + c = (x^3 - 7x + 6) \,.$
$. \,(px + q) + 0$
$= px^4 + qx^3 - 7px^2 - 7qx + 6px + 6q =$
$= px^4 + qx^3 - 7px^2 + (6p - 7q) \,.\, x + 6q$
$p = 2$
$q = -3$
$a = -7p \Rightarrow a = -14$
$b = 6p - 7q \Rightarrow b = 33$
$c = 6q \Rightarrow c = -18$

50) A
Exemplo
$f(x) = x^4$
$g(x) = x^6$
$h(x) = x^3$
$(f + g) \,.\, h = (x^4 + x^6) \,.\, x^3 = x^7 + x^9$
Grau = 9

51) A

$$\begin{array}{l|l} x^m & D(x) \\ \cline{2-2} & x^{m-4} \end{array}$$

$gr(P) = gr(D) + gr(Q)$
$m = gr(D) + m - 4$
$gr(D) = 4$
$gr(R) \leq gr(D) - 1$
$gr(R) \leq 3$

52) A
Exemplo
$p(x) = x^4$
$q(x) = x^5$
$p(x) + q(x) = x^4 + x^5$
Grau = 5

53) 29
01. **Verdadeira**.
$gr(Q) = gr(P) - gr(D) = 10 - 1 = 9$
02. **Falsa**. O resto pode ter grau, no máximo, igual a 17.
04. **Verdadeira**. Teoria.
08. **Verdadeira**.
$gr(Q) = gr(P) - gr(Q) = n + 2 - 2 = n$
16. **Verdadeira**. Teoria.

54) C

$$\begin{array}{r|l} x^3 - 1 & x^2 + 1 \\ \cline{2-2} -x^3 - x & x \\ \hline -x - 1 & \end{array}$$

quociente = x

55)

$$\begin{array}{l|l}
x^5 + 2x^4 + 3x^3 + ax^2 - 4x + 12 & x^3 + 2x^3 - x + 3 \\ \cline{2-2}
-x^5 - 2x^4 + x^3 - 3x^2 & x^2 + 4 \\ \cline{1-1}
4x^3 + (a-3)x^2 - 4x + 12 & \\
-4x^3 - 8x^2 + 4x - 12 & \\ \cline{1-1}
\underbrace{(a-11)x^2}_{} &
\end{array}$$

R(x) = (a – 11) . x² = 0
a – 11 = 0 ⇒ a = 11

56) D

$$\begin{array}{l|l}
\cancel{2x^5} - x^4 + x^2 & 2x + 3 \\ \cline{2-2}
-\cancel{2x^5} - 3x^4 & x^4 - 2x^3 + 3x^2 - 4x + 6 \\ \cline{1-1}
-\cancel{4x^4} + x^2 & \\
\cancel{4x^4} + 6x^3 & \\ \cline{1-1}
\cancel{6x^3} + x^2 & \\
-\cancel{6x^3} - 9x^2 & \\ \cline{1-1}
-\cancel{8x^2} & \\
\cancel{8x^2} + 12x & \\ \cline{1-1}
\cancel{12x} & \\
-\cancel{12x} - 18 & \\ \cline{1-1}
\underbrace{-18}_{} &
\end{array}$$

Q(x) = x⁴ – 2x³ + 3x² – 4x + 6
R(x) = –18

57) B

$$\begin{array}{l|l}
\cancel{x^3} + px + q & x^2 + x + 1 \\ \cline{2-2}
-\cancel{x^3} - x^2 - x & x - 1 \\ \cline{1-1}
-\cancel{x^2} + (p-1)x + q & \\
\cancel{x^2} + x + 1 & \\ \cline{1-1}
\underbrace{px + q + 1}_{} &
\end{array}$$

Q(x) = x – 1

58) D

$$\begin{array}{l|l}
\cancel{x^3} - 2x^2 + 4 & x^2 - 4 \\ \cline{2-2}
-\cancel{x^3} + 4x & x - 2 \\ \cline{1-1}
-\cancel{2x^2} + 4x + 4 & \\
\cancel{2x^2} - 8 & \\ \cline{1-1}
\underbrace{4x - 4}_{} &
\end{array}$$

R(x) = 4x – 4

59) D

$$\begin{array}{l|l}
\cancel{x^3} - 2x^2 + x - 1 & x^2 - x + 1 \\ \cline{2-2}
-\cancel{x^3} + x^2 - x & x - 1 \\ \cline{1-1}
-\cancel{x^2} - 1 & \\
\cancel{x^2} - x + 1 & \\ \cline{1-1}
\underbrace{-x}_{} &
\end{array}$$

R(x) = –x ⇒ R(1) = –1

60) A

$$\begin{array}{l|l}
\cancel{x^4} - 10x^3 + 24x^2 + 10x - 24 & x^2 - 6x + 5 \\ \cline{2-2}
-\cancel{x^4} + 6x^3 - 5x^2 & x^2 - 4x - 5 \\ \cline{1-1}
-\cancel{4x^3} + 19x^2 + 10x - 24 & \\
\cancel{4x^3} - 24x^2 + 20x & \\ \cline{1-1}
-\cancel{5x^2} + 30x - 24 & \\
\cancel{5x^2} - 30x + 25 & \\ \cline{1-1}
\underbrace{1}_{} &
\end{array}$$

Q(x) = x² – 4x – 5 = 0
x' = 5; x '' = –1

61) B

Divisível ⇒ resto = 0

$$
\begin{array}{l|l}
\cancel{x^4} - 4x^3 - 10x^2 + ax + b & x^2 - x + 5 \\ \cline{2-2}
-\cancel{x^4} + x^3 - 5x^2 & x^2 - 3x - 18 \\ \cline{1-1}
-\cancel{3x^3} - 15x^2 + ax + b & \\
\cancel{3x^3} - 3x^2 + 15x & \\ \cline{1-1}
-\cancel{18x^2} + (a + 15)x + b & \\
\cancel{18x^2} - 18x + 90 & \\ \cline{1-1}
\underbrace{(a - 3)x + b + 90}_{} &
\end{array}
$$

R(x) = (a – 3) . x + b + 90 ≡ 0
a – 3 = 0
a = 3
b + 90 = 0
b = –90 vbn
Logo, a + b = 3 – 90 = –87.

62) E

Divisível ⇒ resto = 0

$$
\begin{array}{l|l}
\cancel{x^3} + 2x^2 + mx + n & x^2 + x + 1 \\ \cline{2-2}
-\cancel{x^3} - x^2 - x & x + 1 \\ \cline{1-1}
\cancel{x^2} + (m - 1)x + n & \\
-\cancel{x^2} - x - 1 & \\ \cline{1-1}
\underbrace{(m - 2)x + n - 1}_{} &
\end{array}
$$

Resto = (m – 2) . x + n – 1 ≡ 0
m – 2 = 0
m = 2
n – 1 = 0
n = 1
Logo, m + n = 2 + 1 = 3.

63) C

Divisível ⇒ resto = 0

$$
\begin{array}{l|l}
\cancel{x^4} - 12x^3 + 47x^2 + mx + n & x^2 - 7x + 6 \\ \cline{2-2}
-\cancel{x^4} + 7x^3 - 6x^2 & x^2 - 5x + 6 \\ \cline{1-1}
-\cancel{5x^3} + 41x^2 + mx + n & \\
\cancel{5x^3} - 35x^2 + 30x & \\ \cline{1-1}
+\cancel{6x^2} + (m + 30)x + n & \\
-\cancel{6x^2} + 42x - 36 & \\ \cline{1-1}
\underbrace{(m + 72)x + n - 36}_{} &
\end{array}
$$

R(x) = (m + 72) . x + n – 36 ≡ 0
m + 72 = 0
m = – 72
n – 36 = 0
n = 36
Logo, m + n = – 72 + 36 = – 36

64) B

Divisível ⇒ resto = 0

$$
\begin{array}{l|l}
\cancel{x^3} + 2x^2 + px + q & x^2 + x + 1 \\ \cline{2-2}
-\cancel{x^3} - x^2 - x & x + 1 \\ \cline{1-1}
\cancel{x^2} + (p - 1)x + q & \\
-\cancel{x^2} - x - 1 & \\ \cline{1-1}
\underbrace{(p - 2)x + q - 1}_{} &
\end{array}
$$

R(x) = (p – 2) . x + q – 1 ≡ 0
p – 2 = 0
p = 2
q – 1 = 0
q = 1
Logo, p + q = 2 + 1 = 3.

65) D

Divisível ⇒ resto = 0

$$
\begin{array}{l|l}
\cancel{x^3} + px + q & x^2 + 2x + 5 \\ \cline{2-2}
-\cancel{x^3} - 2x^2 - 5x & x - 2 \\ \cline{1-1}
-\cancel{2x^2} + (p - 5)x + q & \\
\cancel{2x^2} + 4x + 10 & \\ \cline{1-1}
\underbrace{(p - 1)x + q + 10}_{} &
\end{array}
$$

R(x) = (p – 1) . x + q + 10 ≡ 0
p – 1 = 0
p = 1
q + 10 = 0
q = –10

66) Procurando pelas raízes de Q(x), tem-se:
x' = 1 ⇒ $1^3 - 7 . 1 + 6 = 0$ ⇒ 1ª raiz

$$
\begin{array}{c|ccc|c}
1 & 1 & 0 & -7 & 6 \\ \hline
 & 1 & 1 & -6 & 0
\end{array}
$$

Outras raízes
$x^2 + x - 6 = 0$
x'' = –3
x''' = 2
Para que P(x) seja divisível por Q(x), é necessário que x' = 1, x'' = –3 e x''' = 2 sejam raízes de P(x) também.
$P(1) = 2 . 1^4 - 3 . 1^3 - a . 1^2 + b . 1 - c = 0$
$P(-3) = 2 . (-3)^4 - 3 . (-3)^3 - a . (-3)^2 + b . (-3) - c = 0$

$P(2) = 2 . 2^4 - 3 . 2^3 - a . 2^2 + b . 2 - c = 0$

Forma-se o sistema:

$$\begin{cases} -a + b - c = 1 & .(-1) \\ -9a - 3b - c = -243 \\ -4a + 2b - c = -8 \end{cases}$$

$$+\begin{cases} a - b + c = -1 \\ -9a - 3b - c = -243 \end{cases}$$

$-8a - 4b = -244$

$$+\begin{cases} a - b + c = -1 \\ -4a + 2b - c = -8 \end{cases}$$

$-3a + b = 9$

Novo sistema:

$$\begin{cases} -8a - 4b = -244 \\ -3a + b = -9 & .(4) \end{cases}$$

$-3a + b = 9$

$-3 . (14) + b = -9$

$b = -9 + 42$

$b = 33$

$$+\begin{cases} -8a - 4b = -244 \\ -12a + 4b = -36 \end{cases}$$

$-20a = -280$

$a = 14$

$-a + b - c = 1$

$-14 + 33 - c = 1$

$-c = 1 - 19$

$-c = -18$

$c = 18$

$a + b + c = 14 + 33 + 18 = 65$

67) B

Seja $P(x) = a . (x - r_1) . (x - r_2) . (x - r_3)$

Raízes: $x = -2$, $x = 2$, $x = 2$ (gráfico)

Substituindo raízes em P(x):

$P(x) = a . (x + 2) . (x - 2) .(x - 2)$

0 pto $(0, 2) \in P(x)$:

$2 = a . (0 + 2) . (0 - 2) . (0 - 2) \Rightarrow a = \frac{1}{4}$

Substituindo $a = \frac{1}{4}$ em P(x):

$P(x) = \frac{1}{4} . (x + 2) . (x - 2)^2$

$P(x) = \frac{1}{4}x^3 - \frac{1}{2}x^2 - x + 2$

Soma dos coeficientes:

$P(1) = \frac{1}{4} - \frac{1}{2} - 1 + 2 = \frac{3}{4} = 0{,}75$

68) 14

01. **Incorreta**.

$$\left(\frac{1}{3}\right)^{x^2} > \left(\frac{1}{9}\right)^{x-2}$$

$$\left(3^{-1}\right)^{x^2} > \left(3^{-2}\right)^{x-2}$$

$$3^{-x^2} > 3^{-2x+4}$$

$-x^2 > -2x + 4$

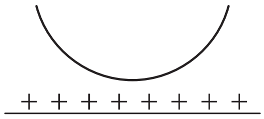
$-x^2 + 2x - 4 > 0$ $\quad .(-1)$

$x^2 - 2x + 4 < 0$

$x^2 - 2x = 4 = 0$

$\Delta = 4 - 16$

$\Delta = -12$



$S = \varnothing$

02. **Correta**.

Verificando suas raízes, obtém-se:

$x' = -1$

$$\begin{array}{r|rrr|r} -1 & 1 & 1 & 4 & 4 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$x^2 + 4$ (Não possui raiz real.)

$x^2 = -4 \notin R$

04. **Correta**.

$2x^3 + 5x^2 - x - 6$

$x - 1 = 0$

$x = 1$

$$\begin{array}{r|rrr|r} 1 & 2 & 5 & -1 & -6 \\ \hline & 2 & 7 & 6 & 0 \end{array}$$

É divisível por $x - 1$.

$$\begin{array}{r|rrr|r} -3/2 & 2 & 5 & -1 & -6 \\ \hline & 2 & 2 & -4 & 0 \end{array}$$

$2x + 3 = 0$

$x = -\frac{3}{2}$

Também é divísivel por $2x + 3$.

08. **Correta**.

sen x = tg x

$\text{sen } x = \frac{\text{sen } x}{\cos x}$

cos x = 1

$x = 2k\pi$

$\text{sen } x = 0 \Rightarrow \text{tg } x = 0$

$x = k\pi$