Matemática C – Semi-Extensivo – V. 2

Exercícios

01) A

$m - 3 \neq 0$
$m \neq 3$
$m^2 - 9 \neq 0$
$m^2 \neq 9$
$m \neq \pm 3$
$m \neq 3$

02) $4x^2 - 4x + 5 = 0$

$\Delta = b^2 - 4ac$
$\Delta = 16 - 4 . 4 . 5$
$\Delta = -64$

$$x = \frac{4 \pm \sqrt{-64}}{8}$$

$$x' = \frac{4 - 8i}{8}$$

$$x'' = \frac{4 + 8i}{8}$$

$$x' = \frac{1}{2} - i$$

$$x'' = \frac{1}{2} + i$$

03) $a + b = 16 \Rightarrow b = 16 - a$

$a . b = 70$
$a . (16 - a) = 70$
$16a - a^2 - 70 = 0 \quad . (-1)$
$a^2 - 16a + 70 = 0$
$\Delta = b^2 - 4ac$
$\Delta = 256 - 280$
$\Delta = -24$

$$a = \frac{16 \pm \sqrt{-24}}{2} \begin{cases} a' = 8 - 2\sqrt{6}\,i \Rightarrow \begin{matrix} b' = 16 - 8 + 2\sqrt{6}\,i \\ b' = -8 + 2\sqrt{6}\,i \end{matrix} \\ a'' = 8 + 2\sqrt{6}\,i \Rightarrow \begin{matrix} b'' = 16 - 8 - 2\sqrt{6}\,i \\ b'' = 8 - 2\sqrt{6}\,i \end{matrix} \end{cases}$$

04) a) $-(y^2 - 16) = 0$

$y^2 - 16 = 0$
$y = \pm\sqrt{16}$
$y' = -4$
$y'' = 4$

b) $x + 6 = 0$

$x = -6$

05) $\begin{cases} 2x + y = 5 \\ x + 4y = 6 \quad . (-2) \end{cases}$ $\quad + \begin{cases} 2x + y = 5 \\ -2x - 8y = -12 \end{cases}$

$0 - 7y = -7$

$y = 1$

$2x + y = 5$
$2x = 5 - 1$
$x = 2$

06) $z = x^2 - 5 + (2 + y)i$

$\overline{w} = 4 + 3i$
$x^2 - 5 = 4$
$x^2 = 9$
$x' = 3$
$x'' = 3$
$2 + y = 3$
$y = 1$

07) C

$x^2 - 4 = 0$
$x^2 = 4$
$x' = 2$
$x'' = -2$

08) $\dfrac{x + 5}{y - 2} + (3x + y)i = 6 + 2yi$

$$\frac{x + 5}{y - 2} = 6$$

$x + 5 = 6y - 12$
$x - 6y = -12 - 5$
$x - 6y = -17$
$3x + y = 2y$
$3x - y = 0$

$$\begin{cases} x - 6y = -17 \quad . (-3) \\ 3x - y = 0 \end{cases}$$

$$+ \begin{cases} -3x + 18y = 51 \\ 3x - y = 0 \end{cases}$$

$17y = 51$
$y = 3$
$3x - y = 0$
$3x - 3 = 0$
$x = 1$

09) C

$i^n = i^m$

$$\frac{i^n}{i^m} = 1$$

$i^{n-m} = 1 \Rightarrow n - m$ é múltiplo de 4.

10) $z = (0, 1)$

$z = i$
$z^{10} = i^{10}$
$z^{10} = i^2$
$z^{10} = -1$
$z^{10} = (-1, 0)$

11) $\frac{i^5 - 2i^{26}}{i^{10}} = \frac{i^1 - 2i^2}{i^{10}} = \frac{i^1 - 2.(-1)}{-1} = -2 - i$

12) C
$n \in C = \{2, 6, 10, 14, 18, ...\}$

13) $i^{250} + i^{104} + 2i^{37} = x + yi$
$i^2 + i^0 + 2i^1 = x + yi$
$-1 + 1 + 2i = x + yi$
$0 + 2i = x + yi$
$x = 0$
$y = 2$

14) $z = a + 2i$
$|z| = \sqrt{a^2 + 2^2} = \sqrt{53}$
$\left(\sqrt{a^2 + 4}\right)^2 = \left(\sqrt{53}\right)^2$
$a^2 + 4 = 53$
$a^2 = 49$
$a' = -7$
$a'' = 7$

15) $\sqrt{a^2 + b^2} = 10$
$a^2 + b^2 = 100$
$a + b = 14$
$b = 14 - a$
$a^2 + b^2 = 100$
$a^2 + (14 - a)^2 = 100$
$a^2 + 196 - 28a + a^2 = 100$
$2a^2 - 28a + 96 = 0$
$a^2 - 14a + 48 = 0$
$a' = 6$
$a'' = 8$

Se $a = 6$, então:
$b = 14 - 6$
$b = 8$
$z = 6 + 8i$

Se $a = 8$, então:
$b = 14 - 8$
$b = 6$
$z = 8 + 6i$

16) $(a + bi)^2 = -3 + 4i$
$a^2 + 2abi + b^2i^2 = -3 + 4i$
$a^2 - b^2 + 2abi = -3 + 4i$
$\begin{cases} a^2 - b^2 = -3 \\ 2ab = 4 \Rightarrow b = \frac{2}{a} \end{cases}$
$a^2 - b^2 = -3$
$a^2 - \left(\frac{2}{a}\right)^2 = -3$
$a^2 - \frac{4}{a^2} = -3$
$\frac{a^4 - 4}{a^2} = \frac{-3a^2}{a^2}$
$a^4 + 3a^2 - 4 = 0$
$x = a^2 \Rightarrow x^2 + 3x - 4 = 0$
$x' = -4$
$x'' = 1$
$a^2 = -4$
$a \notin R$
$a^2 = 1$
$a = \pm 1$
$a' = 1$
$a'' = -1$
$b = \frac{2}{a}$
$b' = 2$
$b'' = -2$
$8|a| + 25|b|$
$8 . 1 + 25 . 2 = 58$

17) $12 . 18 + 12(x + 2)i - 36i - 2i(x + 2)i =$
$= 216 + 12xi + 24i - 36i - 2 . (-1) . x - 2 . (-1) . 2 =$
$= 216 + 12xi - 12i + 2x + 4 =$
$= 220 + 2x + (x - 1) . 12i$
$12(x - 1) = 0$
$x - 1 = 0$
$x = 1$

18) $z_1 = 4 - 3i$
$|z_1| = \sqrt{4^2 + (-3)^2}$
$|z_1| = \sqrt{16 + 9}$
$|z_1| = 5$
$z_3 = i$
$z_3^{38} = i^{38}$
$z_3^{38} = i^2$
$z_3^{38} = -1$
$z_2 = -2i$
$\overline{z}_2 . z_3$
$2i . i$
$2i^2 = 2 . (-1) = -2$
$|z_1| + z_3^{38} - \overline{z}_2 . z_3 =$
$= 5 + (-1) - (-2) = 5 - 1 + 2 = 6$

19) $z = a + bi$
$zi + 2(\overline{z} + z) = |4 + 3i| + i^{23}$
$(a + bi)i + 2(a - bi + a + bi) = \sqrt{4^2 + 3^2} + i^3$
$ai + bi^2 + 2(2a) = 5 - i$
$4a - b + ai = 5 - i$
$a = -1$
$4a - b = 5$
$-4 - b = 5$
$-b = 9$
$b = -9$
$z = -1 - 9i$

20) $\dfrac{x+(x-1)i}{2+i}$

$\dfrac{x+xi-i}{2+i}\cdot\dfrac{(2-i)}{(2-i)} =$

$= \dfrac{2x-xi+2xi-xi^2-2i+i^2}{4-i^2} =$

$= \dfrac{2x+xi+x\,.\,2i-1}{5} =$

$= \dfrac{3x-1+(x-2)i}{5} = \qquad \dfrac{3x-1}{5}$

$\dfrac{3x-1}{5} = 0$

$3x - 1 = 0$

$3x = 1$

$x = \dfrac{1}{3}$

21) $z = a + bi$

$z\,.\,\bar{z} + (z - \bar{z}) = 34 + 10i$

$(a + bi)\,.\,(a - bi) + (a + bi - a + bi) = 34 + 10i$

$a^2 - b^2i^2 + 2bi = 34 + 10i$

$a^2 + b^2 + 2bi = 34 = 10i$

$2b = 10$

$b = 5$

$a^2 + b^2 = 34$

$a^2 + 5^2 = 34$

$a^2 = 34 - 25$

$a = \pm\sqrt{9}$

$a' = 3$

$a'' = -3$

$z' = 3 + 5i$

$z'' = -3 + 5i$

22) A

$z = \dfrac{k-i}{3+i}\cdot\dfrac{3-i}{3-i}$

$z = \dfrac{3k-ki-3i+i^2}{9-i^2}$

$z = \dfrac{3k-1-(k+3)i}{10}$

$|z| = \sqrt{\left(\dfrac{3k-1}{10}\right)^2+\left(-\dfrac{(k+3}{10}\right)^2} = \dfrac{\sqrt{5}}{5}$

$\sqrt{\dfrac{9k^2-\cancel{6k}+1+k^2+\cancel{6k}+9}{100}} = \dfrac{\sqrt{5}}{5}$

$\left(\sqrt{\dfrac{10k^2+10}{100}}\right)^2 = \left(\dfrac{\sqrt{5}}{5}\right)^2$

$\dfrac{10k^2+10}{100} = \dfrac{5}{25}$

$\dfrac{k^2+1}{10} = \dfrac{5}{25}$

$k^2 + 1 = \dfrac{50}{25}$

$k^2 + 1 = 2$

$k^2 = 1$

$k' = 1$

$k'' = -1$

23) $z_1 = a + bi$

$z_2 = c + di$

$\sqrt{a^2+b^2} = 1$

$a^2 + b^2 = 1$

$\sqrt{c^2+d^2} = 1$

$c^2 + d^2 = 1$

$a + bi + c + di = 1$

$a + c = 1$

$a = 1 - c$

$b + d = 0$

$b = -d$

$a^2 + b^2 = c^2 + d^2$

$a^2 + (-d)^2 = c^2 + d^2$

$a^2 + d^2 = c^2 + d^2$

$a^2 = c^2$

$(1 - c)^2 = c^2$

$1 - 2c + c^2 = c^2$

$2c = 1$

$c = \dfrac{1}{2}$

$a = 1 - \dfrac{1}{2}$

$a = \dfrac{1}{2}$

$a^2 + b^2 = 1$

$\dfrac{1}{4} + b^2 = 1$

$b^2 = 1 - \dfrac{1}{4}$

$b = \pm\dfrac{\sqrt{3}}{2}$

$c^2 + d^2 = 1$

$\dfrac{1}{4} + d^2 = 1$

$d^2 = 1 - \dfrac{1}{4}$

$d = \pm\dfrac{\sqrt{3}}{2}$

$z_n = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

$z_2 = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

24) $z = \frac{x - i}{x + i} \cdot \frac{x - i}{x - i}$

$z = \frac{x^2 - 2xi + i^2}{x^2 - i^2}$

$z = \frac{x^2 - 1}{x^2 + 1} - \frac{2xi}{x^2 + 1}$

$\frac{x^2 - 1}{x^2 + 1} < 0$

$x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$ $\Rightarrow$ + + + + ○ – – – – – ○ + + + + (−1, 1)

$x^2 + 1 = 0 \Rightarrow$ + + + + + + + + + + + + + + +
$x^2 = -1$ $\Rightarrow$ + + + + ○ – – – – – ○ + + + + (−1, 1)
$x \notin R$
$S = \{x \in R / -1 < x < 1\}$

25) $z = \frac{2}{i} + \frac{2}{3 - i} - \frac{x}{3 + i}$

$z = \frac{2}{i}\frac{i}{i} + \frac{2}{3 - i} \cdot \frac{3 + i}{3 + i} - \frac{x}{3 + i} \cdot \frac{3 - i}{3 - i}$

$z = \frac{2i}{-1} + \frac{6 + 2i}{9 - i^2} - \frac{3x - xi}{9 - i^2}$

$z = -2i + \frac{6 + 2i}{10} - \frac{3x - xi}{10}$

$z = \frac{-20i + 6 + 2i - 3x + xi}{10}$

$z = \frac{6 - 3x}{10} + \frac{(-18 + x)i}{10}$

$\frac{6 - 3x}{10} = 0$
$6 - 3x = 0$
$3x = 6$
$x = 2$

26) $\begin{cases} a_1 + a_3 = 1 - i \\ a_2 + a_4 = 2i \end{cases}$

$a_1 + a_1 \cdot q^2 = 1 - i$
$a_1 \cdot q + a_1 \cdot q^3 = 2i$

$\frac{\cancel{a_1} \cdot q(1 + \cancel{q^2})}{\cancel{a_1}(1 + \cancel{q^2})} = \frac{2i}{1 - i}$

$q = \frac{2i}{1 - i}$

$q = \frac{2i}{1 - i} \cdot \frac{1 + i}{1 + i}$

$q = \frac{-2 + 2i}{1 - i^2}$

$q = -1 + i$

27) $\begin{vmatrix} 1 & i & 1 \\ i & 1 & i \\ 1 + i & 1 - i & 0 \end{vmatrix} \begin{matrix} 1 & i \\ i & 1 \\ 1 + i & 1 - i \end{matrix}$

$0 + i^2 \cdot (1 + i) + i \cdot (1 - i) - 1 - i - i \cdot (1 - i) - 0 =$
$= \cancel{i^2} + i^3 + \cancel{i} - \cancel{i^2} - 1 - \cancel{i} - i + i^2 =$
$= i^3 + i^2 - i - 1 =$
$= -i - 1 - i - 1$
$z = -2 - 2i$

$|z| = \sqrt{(-2)^2 + (-2)^2}$

$|z| = \sqrt{8}$

$|z| = 2\sqrt{2}$

28) $z = a + bi$
$(1 + i) \cdot z - (1 + 2i) \cdot \bar{z} = 7 + 3i$
$(1 + i) \cdot (a + bi) \cdot (1 + 2i) \cdot (a - bi) = 7 + 3i$
$\cancel{a} + bi + ai + bi^2 - \cancel{a} + bi - 2ai + 2bi^2 = 7 + 3i$
$3bi^2 + 2bi - ai = 7 + 3i$
$-3b + (2b - a)i = 7 + 3i$
$-3b = 7$

$b = -\frac{7}{3}$

$2b - a = 3$

$2 \cdot \left(-\frac{7}{3}\right) - a = 3$

$a = -\frac{14}{3} - 3$

$a = -\frac{23}{3}$

$a + b = -\frac{7}{3} - \frac{23}{3}$

$a + b = -\frac{30}{3}$

$a + b = -10$

29) C
$z_1 \cdot z_2 = 15$
$(a + bi) \cdot (1 - 2i) = 15$
$a + bi - 2ai - 2bi^2 = 15$
$a + 2b + (-2a + b)i = 15$

$\begin{cases} a + 2b = 15 & .(2) \\ -2a + b = 0 \end{cases}$

$+\begin{cases} 2a + 4b = 30 \\ -2a + b = 0 \end{cases}$

$5b = 30$
$b = 6$

$a + 2b = 15$
$a + 12 = 15$
$a = 3$
$z_1 + z_2 = 3 + 6i + 1 - 2i = 4 + 4i$

30) $z = x + yi$

a) $2\bar{z} - i + \bar{z} =$
$= 3z - i =$
$= 3(x + yi) - i =$
$= 3x + (3y - 1)i$
Parte real: $3x$
Parte imaginária: $3y - 1$

b) $2z - i + \bar{z} = 0$
$2(x + yi) - i + x - yi = 0$
$2x + 2yi - i + x - yi = 0$
$3x + (y - 1)i = 0$
$3x = 0$
$x = 0$
$y - 1 = 0$
$y = 1$
$z = 0 + 1i$
$z = i$

31) $z = \dfrac{(10 - i)i^3 + i^{50}}{(1 - i)^2}$

$z = \dfrac{(10 - i)\,.\,(-1) + i^2}{1 - 2i + i^2}$

$z = \dfrac{-10i + i^2 + i^2}{-2i}$

$z = \dfrac{-2 - 10i}{-2i}$

$z = \dfrac{-1 - 5i}{-i} \cdot \dfrac{-i}{-i}$

$z = \dfrac{i + 5i^2}{i^2}$

$z = \dfrac{-5 + i}{-1}$

$z = 5 - i$

$|z| = \sqrt{5^2 + (-1)^2}$

$|z| = \sqrt{26}$

$|z|^2 = 26$

32) $z + \bar{z} = 18$
$z\,.\,\bar{z} = -18$
$(a + bi) + (a - bi) = 18$
$2a = 18$
$a = 9$
$z\,.\,\bar{z} = -18$
$(a + bi)\,.\,(a - bi) = -18$
$(a + bi)\,.\,(9 - bi) = -18$
$81 - b^2i^2 = -18$
$-b^2\,.\,(-1) = -99$
$b^2 = 99$
$b = 3\sqrt{11}$
$a\,.\,b = 9\,.\,3\sqrt{11} = 27\sqrt{11}$

33) $f(z) = 2z^2 + 4z + 5$
$f(z) = 2(i - 1)^2 + 4(i - 1) + 5$
$f(z) = 2(i^2 - 2i + 1) + 4i - 4 + 5$
$f(z) = 2i^2 - \cancel{4i} + 1 + \cancel{4i} + 1$
$f(z) = 2(-1) + 2$
$f(z) = 0$

34) $z = i(a + 3i)$
$z = ai + 3i^2$
$z = -3 + ai$

$|z| = \sqrt{(-3)^2 + a^2} = 5$

$\left(\sqrt{9 + a^2}\right)^2 = (5)^2$

$9 + a^2 = 25$
$a^2 = 25 - 9$
$a = \pm\sqrt{16}$
$a' = +4$
$a'' = -4$

35) $2z + \bar{z} = 2zi - z$
$2(a + bi) + (a - bi) = 2(a + bi)i - (a + bi)$
$2a + 2bi + a - bi = 2ai + 2bi^2 - a - bi$
$3a + bi = 2ai - 2b - a - bi$
$3a + bi - 2ai + 2b + a + bi = 0$
$4a + 2b + 2bi - 2ai = 0$
$4a + 2b + 2(b - a)i = 0$

$$\begin{cases} 4a + 2b = 0 \\ -a + b = 0 \quad .\,(4) \end{cases}$$

$$+\begin{cases} 4a + 2b = 0 \\ -4a + 4b = 0 \end{cases}$$

$6b = 0$
$b = 0$
$-a + b = 0$
$-a + 0 = 0$
$a = 0$

36) $z = 10\,.\left(\cos\dfrac{\pi}{6} + i\operatorname{sen}\dfrac{\pi}{6}\right)$

$z = 10\,.\left(\dfrac{\sqrt{3}}{2} + i\,.\,\dfrac{1}{2}\right)$

$z = 5\sqrt{3} + 5i$

$a^2 - b^2 = (5\sqrt{3})^2 - (5)^2$
$a^2 - b^2 = 25\,.\,3 - 25$
$a^2 - b^2 = 50$

37) $z = -\sqrt{2} + i\sqrt{2}$

$|z| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$

$|z| = \sqrt{4}$
$|z| = 2$

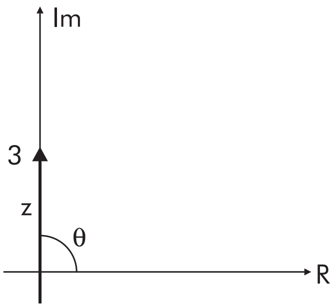
$\cos\ \theta = -\dfrac{\sqrt{2}}{2}$

$\text{sen}\ \theta = \dfrac{\sqrt{2}}{2}$

$\theta = 135°$

$\dfrac{\sqrt{2}}{2} = \dfrac{3\pi}{4}$

38) 08
01. **Falsa**. Se $z = 0$, não existe z^{-1}.
02. **Falsa**.
$z + \bar{z} = a + bi + a - bi = 2a + 0 \Rightarrow$ parte imaginária nula
04. **Falsa**. Pela representação no plano, temos:

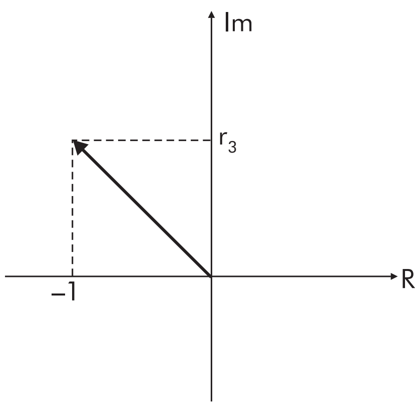


módulo = 3

Porém, $\theta = \dfrac{\pi}{2}$.

08. **Verdadeira**.
$z = 2i$
$z^6 = (2i)^6$
$z^6 = 2^6 \, .\, i^6$
$z^6 = 64\, .\, i^2$
$z^6 = -64$

39) $z = -1 + \sqrt{3}$



$|z| = \sqrt{(-1)^2 + (\sqrt{3})^2}$

$|z| = \sqrt{1 + 3}$

$|z| = 2$

$\text{sen}\,\theta = \dfrac{\sqrt{3}}{2}$

$\theta = 120°$

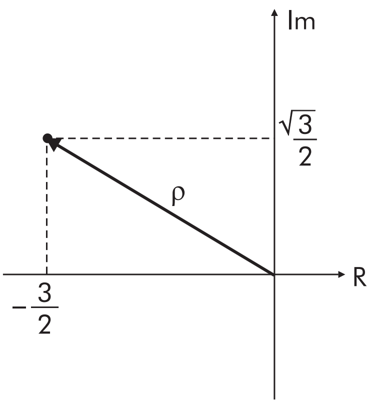
40) $z = \sqrt{18}\ (\cos 315° + i\text{sen } 315°)$

$z = \sqrt{18}\left(+\dfrac{\sqrt{2}}{2} + i\left(-\dfrac{\sqrt{2}}{2}\right)\right)$

$z = \dfrac{\sqrt{36}}{2} - \dfrac{\sqrt{36}}{2}\,i$

$z = 3 - 3i$
$3 - 3 = 0$

41)



$\rho^2 = \left(-\dfrac{3}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2$

$\rho^2 = \dfrac{9}{4} + \dfrac{3}{4}$

$\rho^2 = 3$

$\rho^2 = \sqrt{3}$

$\text{sen}\,\theta = \dfrac{\frac{\sqrt{3}}{2}}{\sqrt{3}}$

$\text{sen}\,\theta = \dfrac{1}{2}$

$\theta = 150°$

$\theta = \dfrac{2\pi}{3}$

$z = \sqrt{3}\left(\cos\dfrac{2\pi}{3} + i\text{sen}\,\dfrac{2\pi}{3}\right)$

42) $z = \frac{4i}{1+i} \cdot \frac{1-i}{1-i}$

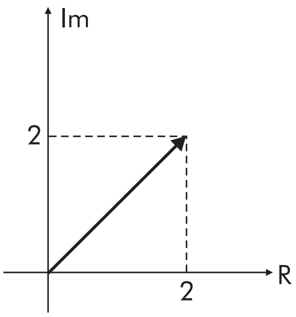
$z = \frac{4i - 4i^2}{1 - i^2}$

$z = \frac{4 + 4i}{2}$

$z = 2 + 2i$

$|z| = \sqrt{2^2 + 2^2}$

$|z| = \sqrt{8}$



$\text{sen}\,\theta = \frac{2}{\sqrt{8}}$

$\text{sen}\,\theta = \frac{\sqrt{2}}{2}$

$\theta = 45°$

$\theta = \frac{\pi}{4}$

$z = \sqrt{8}\left(\cos\frac{\pi}{4} + i\,\text{sen}\,\frac{\pi}{4}\right)$

43) A

$|z| = \sqrt{(x-2)^2 + (x+3)^2} = 5$

$(x-2)^2 + (x+3)^2 = 25$

$x^2 - 4x + 4 + x^2 + 6x + 9 = 25$

$2x^2 + 2x + 13 - 25 = 0$

$x^2 + x - 6 = 0$

~~$x' = -3$~~

$x'' = 2$

$z = (2 . 2) + (2 + 3)i$

$z = 5i$

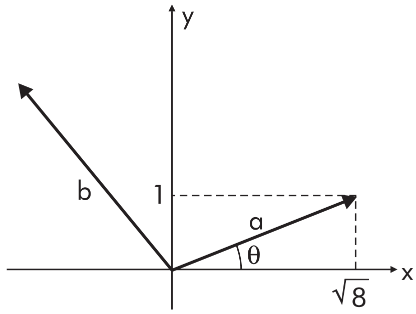
44) $\text{sen}\,\frac{n\pi}{3} = 0$

$\frac{n\pi}{3} = 0 + k\pi$

$\begin{cases} \frac{n\pi}{3} = 0 \Rightarrow n = 0 \\ \frac{n\pi}{3} = \pi \Rightarrow n = 3 \end{cases}$

$S = 3$

45)



$|a| = \sqrt{(\sqrt{8})^2 + 1^2}$

$|a| = \sqrt{9}$

$|a| = 3$

$\text{sen}\,\theta = \frac{1}{3}$

$\text{sen}\,(\theta + 90°) = \frac{\sqrt{8}}{3}$

$\text{sen}\,(\theta + 90°) = \frac{2\sqrt{2}}{3}$

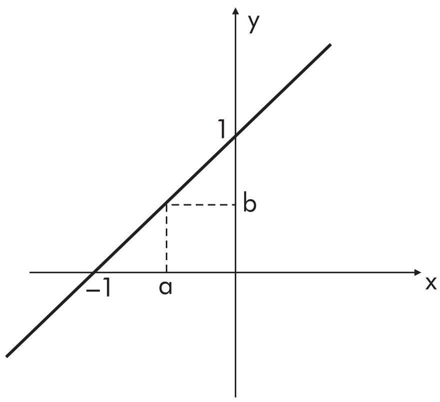
$\cos\,\theta = \frac{\sqrt{8}}{3}$

$\cos\,(\theta + 90°) = -\frac{1}{3}$

$b = 1(\cos\,\theta + i\,\text{sen}\,\theta)$

$b = -\frac{1}{3} + \frac{2\sqrt{2}}{3}i$

46)



$z = x + iy$

$y = x + 1$

$z = x + i(x + 1)$

$|z| = \sqrt{x^2 + (x+1)^2} = \sqrt{13}$

$x^2 + x^2 + 2x + 1 = 13$

$2x^2 + 2x - 12 = 0$

$x^2 + x - 6 = 0$

$x' = 2$

$x'' = -3$

$y = x + 1$

$y' = 2 + 1$

$y' = 3$

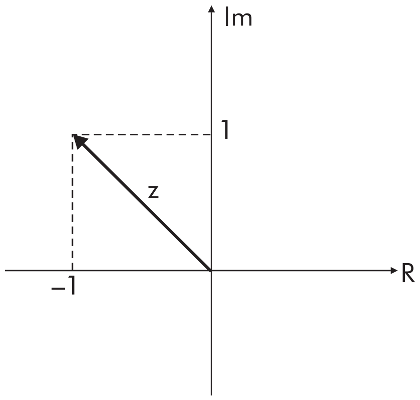
$z = 2 + 3i$
ou
$y'' = -3 + 1$
$y'' = -2$
$z = -3 - 2i$

47) $iz + 2\bar{z} + 1 - i = 0$
$i(a + bi) + 2(a - bi) + 1 - i = 0$
$ai + bi^2 + 2a - 2bi + 1 - i = 0$
$ai - b + 2a - 2bi + 1 - i = 0$

$$\begin{cases} 2a - b + 1 = 0 \quad . (-2) \\ a - 2b - 1 = 0 \end{cases}$$

$$+\begin{cases} -4a + 2b - 2 = 0 \\ a - 2b - 1 = 0 \end{cases}$$
$$-3a + 0 - 3 = 0$$

$-3a = 3$
$a = -1$
$a - 2b - 1 = 0$
$-1 - 2b - 1 = 0$
$-2b = 2$
$b = -1$
$z = -1 - 1i$



$|z| = \sqrt{(-1)^2 + (-1)^2}$

$|z| = \sqrt{2}$

$\text{sen}\,\theta = -\dfrac{1}{\sqrt{2}}$

$\text{sen}\,\theta = -\dfrac{\sqrt{2}}{2}$

$\theta = 225°$

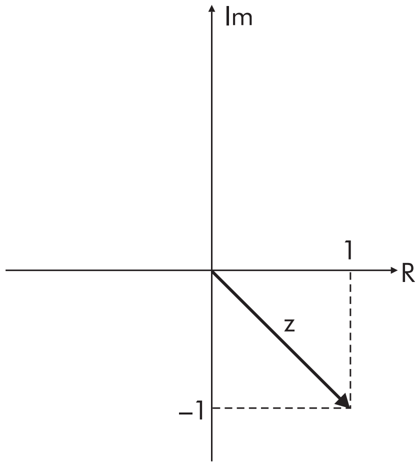
48) $z_1 = \dfrac{1}{3} - \dfrac{2}{5}i$

$z_2 = -\dfrac{2}{3} - \dfrac{3}{5}i$

$z_1 - z_2 = \dfrac{1}{3} - \dfrac{2}{5}i + \dfrac{2}{3} + \dfrac{3}{5}i$

$z_1 - z_2 = \dfrac{3}{3} - \dfrac{5}{5}i$

$z_1 - z_2 = 1 - i$



$|z_1 - z_2| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\cos\theta = \dfrac{1}{\sqrt{2}}$

$\cos\theta = \dfrac{\sqrt{2}}{2}$

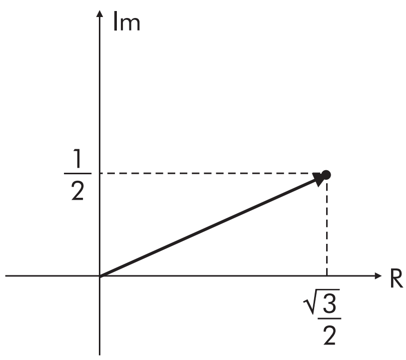
$\theta = 315°$

$\theta = \dfrac{7\pi}{4}$

$\theta = -\dfrac{\pi}{4}$

$z_1 - z_2 = \sqrt{2}\left(\cos\left(-\dfrac{\pi}{4}\right) + i\,\text{sen}\left(-\dfrac{\pi}{4}\right)\right)$

49)



$|\mu| = \sqrt{\left(\dfrac{\sqrt{3}}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2}$

$|\mu| = \sqrt{\dfrac{3}{4} + \dfrac{1}{4}}$

$|\mu| = \sqrt{1}$

$|\mu| = 1$

$\text{sen}\,\theta = \dfrac{\frac{1}{2}}{1}$

$\text{sen}\,\theta = \frac{1}{2}$

Pelo desenho, temos:

$\theta = 30°$

$\theta = \frac{\pi}{6}$

$|v| = 2$

argumento de v = 90°

$v = 2(\cos 90° + i\,\text{sen}\,90°)$

$v = 2(0 + i)$

$v = 2i$

50) $z \cdot \bar{z} = 25$

$(a + bi) \cdot (a - bi) = 25$

$a^2 - b^2i^2 = 25$

$a^2 + b^2 = 25$

$|z| = \sqrt{a^2 + b^2}$

$|z| = \sqrt{25}$

$|z| = 5$

51) $\frac{z}{w} = \frac{12}{2}(\cos(40 - 10) + i\,\text{sen}\,(40 - 10))$

$\frac{z}{w} = 6(\cos 30° + i\,\text{sen}\,30°)$

$\frac{z}{w} = 6\left(\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}\right)$

$\frac{z}{w} = 3\sqrt{3} + 3i$

52) $z^{10} = 2^{10}(\cos(10 \cdot 30) + i\,\text{sen}\,(10 \cdot 30))$

$z^{10} = 1024(\cos 300° + i\,\text{sen}\,300°)$

$z^{10} = 1024\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$z^{10} = 512 - 512\sqrt{3}\,i$

53) $z = \sqrt{3} + i$

$|z| = \sqrt{(\sqrt{3})^2 + 1^2}$

$|z| = \sqrt{3 + 1}$

$|z| = 2$

$\left.\begin{matrix} \text{sen}\,\theta = \frac{1}{2} \\ \cos\theta = \frac{\sqrt{3}}{2} \end{matrix}\right\rangle \theta = 30°$

$z = 2(\cos 30° + i\,\text{sen}\,30°)$

$z^n = 2^n(\cos(30 \cdot n) + i\,\text{sen}\,(30 \cdot n))$

Para ser real:

sen (30 n) = 0

30 n = 0 + k . 180

1º) $30n = 0 \Rightarrow$ ~~n = 0~~

2º) $30n = 180 \Rightarrow n = 6$

54) D

$\frac{\mu}{v} = \frac{\rho}{3}(\cos(\alpha - 60) + i\,\text{sen}\,(\alpha - 60)) =$

$= 2(\cos 30° + i\,\text{sen}\,30°)$

$\frac{\rho}{3} = 2$

$\rho = 6$

$\cos(\alpha - 60) = \cos 30$

$\alpha - 60 = 30$

$\alpha = 90°$

$\mu = 6(\cos 90° + i\,\text{sen}\,90°)$

$\mu = 6(0 + i \cdot 1)$

$\mu = 6i$

55) $E = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^8 \cdot \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^8$

Escrevendo a forma trigonométrica dos números complexos, encontramos:

$z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$z_1 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$z_1 = \sqrt{\frac{1 + 3}{4}}$

$z_1 = 1$

$\text{sen}\,\theta = \frac{\frac{\sqrt{3}}{2}}{1}$, $\text{sen}\,\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\frac{1}{2}}{1}$, $\cos\theta = \frac{1}{2}$

$\theta = 60°$

$z_1 = (\cos 60° + i\,\text{sen}\,60°)$

$(z_1)^8 = (\cos 480° + i\,\text{sen}\,480°)$

$(z_1)^8 = (\cos 120° + i\,\text{sen}\,120°)$

$z_2 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$|z_2| = 1$

$\text{sen}\,\theta = \frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30°$

$z_2 = (\cos 30° + i\,\text{sen}\,30°)$

$(z_2)^8 = (\cos 240° + i\,\text{sen}\,240°)$

$E = (z_1)^8 \cdot (z_2)^8$
$E = (\cos 120° + i\,\text{sen } 120°) \cdot (\cos 240° + i\,\text{sen } 240°)$
$E = (\cos 360° + i\,\text{sen } 360°)$
$E = 1 + i \cdot 0$
$E = 1$

56) $z^5 = 2^{-5} (\cos (-5 \cdot 50) + i\,\text{sen } (-5 \cdot 50))$

$$z^5 = \frac{1}{32} (\cos -250° + i\,\text{sen } -250°)$$

$$z^5 = \frac{1}{32} (\cos 110° + i\,\text{sen } 110°)$$

57) $z = \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i\right)^n$

$$|z| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$|z| = \sqrt{\frac{3}{4} + \frac{1}{4}}$$

$$|z| = 1$$

$$\text{sen } \theta = \frac{\frac{1}{2}}{1}$$

$$\text{sen } \theta = \frac{1}{2}$$

$$\cos \theta = \frac{\frac{\sqrt{3}}{2}}{1}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$\theta = 30°$
$z = 1(\cos 30° + i\,\text{sen } 30°)$
$z^n = 1^n(\cos (30 \cdot n) + i\,\text{sen } (30 \cdot n))$
$\cos (30 n) = 0$
$30 n = 90° + k \cdot 180$
1º) $30 n = 90°$
$n = 3$

58) B
$z_1 \cdot z_2 \cdot z_3 =$
$= 2 \cdot 3 \cdot 1(\cos (40 + 135 + 125) +$
$+ i\,\text{sen } (40 + 135 + 125)) =$
$= 6(\cos 300° + i\,\text{sen } 300) =$

$$= 6\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = 3 - 3\sqrt{3}\,i$$

59) A

$$\frac{\pi}{4} = 45°$$

$z^7 = (\sqrt{2})^7(\cos (7 \cdot 45°) + i\,\text{sen } (7 \cdot 45°))$

$z^7 = 8\sqrt{2}(\cos 315° + i\,\text{sen } 315°)$

$$z^7 = 8\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right)$$

$z^7 = 8 - 8i$

60) D
$z^{109} = ?$

$$|z| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2}$$

$$|z| = \sqrt{\frac{4}{4}}$$

$$|z| = 1$$

$$\text{sen } \theta = \frac{\frac{\sqrt{2}}{2}}{1}$$

$$\text{sen } \theta = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\frac{\sqrt{2}}{2}}{1}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$\theta = 45°$
$z = 1(\cos 45° + i\,\text{sen } 45°)$
$z^{109} = 1^{109}(\cos (109 \cdot 45) + i\,\text{sen } (109 \cdot 45))$
$z^{109} = \cos (4905) + i\,\text{sen } (4905)$
$z^{109} = \cos 225° + i\,\text{sen } 225°$

$$z^{109} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$$

61) B

$$z^n + \frac{1}{z^n}$$

$z^n = 1^n(\cos (n\theta) + i\,\text{sen } (n\theta)) =$
$= \cos (n\theta) + i\,\text{sen } (n\theta) +$

$$\left[\frac{1}{\cos (n\theta) + i\,\text{sen } (n\theta)} \cdot \frac{\cos (n\theta) - i\,\text{sen } (n\theta)}{\cos (n\theta) - i\,\text{sen } (n\theta)}\right] =$$

$$= \cos (n\theta) + i\,\text{sen } (n\theta) + \left[\frac{\cos (n\theta) - i\,\text{sen } (n\theta)}{\cos^2 (n\theta) - i^2\text{sen}^2 (n\theta)}\right] =$$

$$= \cos (n\theta) + i\,\text{sen } (n\theta) + \left[\frac{\cos (n\theta) - i\,\text{sen } (n\theta)}{\cos^2 (n\theta) + \text{sen}^2 (n\theta)}\right] =$$

$$= \cos (n\theta) + i\,\text{sen } (n\theta) + \frac{\cos (n\theta) - i\,\text{sen } (n\theta)}{1} =$$

$$= 2\cos (n\theta)$$

62) A

$z_1 = 1 + i \Rightarrow$ forma trigométrica

$|z_1| = \sqrt{1^2 + 1^2}$

$|z_1| = \sqrt{2}$

$\text{sen}\,\theta = \frac{1}{\sqrt{2}}$

$\text{sen}\,\theta = \frac{\sqrt{2}}{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = 45°$

$z_2 = 1 - i \Rightarrow$ forma trigométrica

$|z_2| = \sqrt{1^2 + (-1)^2}$

$|z_2| = \sqrt{2}$

$\text{sen}\,\theta = -\frac{1}{\sqrt{2}}$

$\text{sen}\,\theta = -\frac{\sqrt{2}}{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = -45°$

$(z_1)^n = (z_2)^n$
$[1(\cos 45° + i\text{sen}\, 45°)]^n = [1(\cos -45° + i\text{sen} -45°)]^n$
$\cos 45°n + i\text{sen}\, 45°n = \cos -45°n + i\text{sen} -45°n$
$\cos 45°n = \cos -45°n$
$\text{sen}\, 45°n = \text{sen} -45°n$
$n = 4k, k \in Z$

63) $z = 2\left(\cos\frac{\pi}{3} + i\text{sen}\,\frac{\pi}{3}\right)$

$z^6 - 2z^3$

$2^6\left(\cos 6 \cdot \frac{\pi}{3} + i\text{sen}\, 6 \cdot \frac{\pi}{3}\right) -$

$- 2 \cdot 2^3\left(\cos 3 \cdot \frac{\pi}{3} + i\text{sen}\, 3 \cdot \frac{\pi}{3}\right) =$

$= 64(\cos 2\pi + i\text{sen}\, 2\pi) - 16(\cos\pi + i\text{sen}\,\pi) =$
$= 64(1 + 0) - 16(-1 + 0) = 64 + 16 = 80$

64) A

$z^6 = (\sqrt{2})^6\left(\cos 6 \cdot \frac{\pi}{6} + i\text{sen}\, 6 \cdot \frac{\pi}{6}\right)$

$z^6 = 2^3 (\cos\pi + i\text{sen}\,\pi)$
$z^6 = 8(-1 + 0)$
$z^6 = -8$

65) A
$z = 1 - i$

$|z| = \sqrt{1^2 + (-1)^2}$

$|z| = \sqrt{2}$

$\text{sen}\,\theta = -\frac{1}{\sqrt{2}}$

$\text{sen}\,\theta = -\frac{\sqrt{2}}{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = 135°$

$z = \sqrt{2}(\cos 135° + i\text{sen}\, 135°)$

$z^{10} = (\sqrt{2})^{10}(\cos 10 \cdot 135° + i\text{sen}\, 10 \cdot 135°)$
$z^{10} = 2^5(\cos 1350° + i\text{sen}\, 1350°)$
$z^{10} = 2^5(\cos 270° + i\text{sen}\, 270°)$
$z^{10} = 32 \cdot (0 + i \cdot (-1))$
$z^{10} = -32i$