Matemática E – Extensivo – V. 6

Resolva

Aula 21

21.01) $P(x) = 2x^3 + 3x^2 - x + 2$
a) $P(1) = 2 + 3 - 1 + 2 = 6$
b) $P(0) = 0 + 0 - 0 + 2 = 2$
c) $P(-2) = -16 + 12 + 2 + 2 = 0$
d) Grau = 3

21.02) $P(x) = (a - 3)x^3 + (b + 2a)x^2 + (6b + c)x$
$P(x) \equiv 0$
$\Rightarrow a - 3 = 0$
$a = 3$
$b + 2a = 0$
$b + 6 = 0$
$b = -6$
$6b + c = 0$
$-36 + c = 0$
$c = 36$
$\Rightarrow 2 . (a + b + c)$
$2 . (3 - 6 + 36) = 66$

21.03) $P_1(x) = a(x + c)^3 + b(x + d)$
$= a . (x^3 + 3x^2c + 3xc^2 + c^3) + bx + bd$
$= ax^3 + 3acx^2 + 3ac^2x + ac^3 + bx + bd$
$= ax^3 + 3acx^2 + (3ac^2 + b)x + ac^3 + bd$
$P_2(x) = x^3 + 6x^2 + 15x + 14$
$P_1(x) \equiv P_2(x)$
$\Rightarrow a = 1$
$3ac = 6$
$c = 2$
$3ac^2 + b = 15$
$12 + b = 15$
$b = 3$
$ac^3 + bd = 14$
$8 + 3d = 14$
$3d = 6$
$d = 2$

Aula 22

22.01) $A(x) = 4x^3 + 2x - 3$
$B(x) = 2x^2 + 1$
$C(x) = 5 + x - 2x^2 - 4x^3$
a) $A(x) + B(x)$
$= 4x^3 + 2x - 3 + 2x^2 + 1$
$= 4x^3 + 2x^2 + 2x - 2$
b) $A(x) + B(x) + C(x)$
$= \cancel{4x^3} + 2x - 3 + \cancel{2x^2} + 1 + 5 + x - \cancel{2x^2} - \cancel{4x^3}$
$= 3x + 3$
c) $A(x) . B(x)$
$= (4x^3 + 2x - 3) . (2x^2 + 1)$
$= 8x^5 + 4x^3 + 4x^3 + 2x - 6x^2 - 3$
$= 8x^5 + 8x^3 - 6x^2 + 2x - 3$
d) $[B(x)]^3 = [2x^2 + 1]^3$
$= (2x^2)^3 + 3 . (2x^2)^2 + 3 . (2x^2)^1 + 1$
$= 8x^6 + 12x^4 + 6x^2 + 1$

22.02) $P(x) = 2x^3 + 7x^2 - 3x$
$D(x) = x^2 + 3x - 1$
$P(x) \equiv D(x) . Q(x) + R(x)$
$2x^3 + 7x^2 - 3x \equiv (x^2 + 3x - 1) . (ax + b) + cx + d$
$2x^3 + 7x^2 - 3x \equiv ax^3 + bx^2 + 3ax^2 + 3bx - ax - b + cx + d$
$2x^3 + 7x^2 - 3x \equiv ax^3 + (3a + b)x^2 + (3b - a + c)x - b + d$
$\Rightarrow a = 2$
$3a + b = 7 \Rightarrow b = 1$
$3b - a + c = -3 \Rightarrow c = -4$
$-b + d = 0 \Rightarrow d = 1$
$Q(x) = 2x + 1$; $R(x) = -4x + 1$

22.03) E
$P_1(x) = 2x^2 - 3x + 1$
$P_2(x) = 3x - 1$
$P_3(x) = 6x^3 - 11x^2 + 6x - 1$
Dividindo $P_3(x)$ por $P_1(x)$, temos:
$(6x^3 - 11x^2 + 6x - 1) =$
$= (2x^2 - 3x + 1) . (ax + b) + cx + d$
$= 2ax^3 + 2bx^2 - 3ax^2 - 3bx + ax + b + cx + d$
$= 2ax^3 + (2b - 3a)x^2 + (a - 3b + c)x + b + d$
$\Rightarrow 2a = 6 \Rightarrow a = 3$
$2b - 3a = -11 \Rightarrow b = -1$
$a - 3b + c = 6 \Rightarrow c = 0$
$b + d = -1$
$d = 0$
Logo, $R(x) = 0$, e $P_3(x)$ é divisível por $P_1(x)$.

Aula 23

23.01) A

$$\begin{array}{r|l} x^5 + 2x^3 - 3x^2 + 6x - 5 & x^3 + x^2 + 1 \\ \cline{2-2} -x^5 - x^4 - x^2 & x^2 - x + 3 \\ \cline{1-1} -x^4 + 2x^3 - 4x^2 + 6x - 5 & \\ x^4 + x^3 + x & \\ \cline{1-1} 3x^3 - 4x^2 + 7x - 5 & \\ -3x^3 - 3x^2 - 3 & \\ \cline{1-1} -7x^2 + 7x - 8 & \end{array}$$

Logo, $Q(x) = x^2 - x + 3$.

23.02) C

$x^3 - 1 \;|\; x^2 + 1$

$-x^3 - x \quad\quad x$

$-x - 1$

Quociente = x

23.03) $x^5 + 2x^4 + 3x^3 + ax^2 - 4x + 12 \;|\; x^3 + 2x^3 - x + 3$

$-x^5 - 2x^4 + x^3 - 3x^2 \quad\quad x^2 + 4$

$4x^3 + (a-3)x^2 - 4x + 12$

$-4x^3 - 8x^2 + 4x - 12$

$(a - 11)x^2$

$R(x) = (a - 11) \cdot x^2 = 0$

$\Rightarrow a - 11 = 0$

$a = 11$

23.04) $3x^4 - 2x^3 - x^2 + 5x + 1 \;|\; x^2 - 3x + 1$

$-3x^4 + 9x^3 - 3x^2 \quad\quad 3x^2 + 7x + 17$

$7x^3 - 4x^2 + 5x + 1$

$-7x^3 + 21x^2 - 7x$

$17x^2 - 2x + 1$

$-17x^2 + 51x - 17$

$49x - 16$

$Q(x) = 3x^2 + 7x + 17$

$R(x) = 49x - 16$

Aula 24

24.01) E

$$\text{Resto} = P\left(-\frac{1}{2}\right)$$

$$= \left(-\frac{1}{2}\right)^5 - 2 \cdot \left(-\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^2 + 1$$

$$= \frac{-1}{32} - \frac{1}{8} + \frac{1}{4} + 1$$

$$= \frac{-1 - 4 + 8 + 32}{32} = \frac{35}{32}$$

24.02) $P(-3) = 43$

$-27 + 9m + 6 + 1 = 43$

$9m = 63$

$m = 7$

24.03) $P(-2) = 5$

$P(2) = 13$

$P(x) \;|\; x^2 - 4$

$ax + b \quad\quad Q(x)$

$P(x) = (x^2 - 4) \cdot Q(x) + ax + b$

$P(-2) = 5 \Rightarrow \cancel{(4 - 4) \cdot Q(-2)} - 2a + b = 5$

$-2a + b = 5$

$P(2) = 13 \Rightarrow \cancel{(4 - 4) \cdot Q(2)} + 2a + b = 13$

$2a + b = 13$

$\begin{cases} -\cancel{2a} + b = 5 \\ \cancel{2a} + b = 13 \end{cases} \oplus$

$2b = 18$

$b = 9;\ a = 2$

$\Rightarrow R(x) = 2x + 9$

$R(1) = 2 + 9 = 11$

Testes

Aula 21

21.01) $P(x) = 4x^3 + 5x^2 - 7x - 2$
a) $P(1) = 4 + 5 - 7 - 2 = 0$
b) $P(-2) = -32 + 20 + 14 - 2 = 0$
c) $P(0) = 0 + 0 - 0 - 2 = -2$

21.02) $P(x) = mx^2 - 5x + 2$
$P(-2) = 5$
$\Rightarrow 4m + 10 + 2 = 5$
$4m = -7$
$m = -\frac{7}{4}$

21.03) $P(x) = 2x^3 + px^2 - 5x + q$
$P(1) = 7 \Rightarrow 2 + p - 5 + q = 7$
$P(2) = 25 \Rightarrow 16 + 4p - 10 + q = 25$

$\begin{cases} p + q = 10 \quad .(-1) \\ 4p + q = 19 \end{cases}$

$\begin{cases} -p - q = -10 \\ 4p + q = 19 \end{cases} \oplus$

$3p = 9$
$p = 3; q = 7$

21.04) $P(x) = x^3 + ax^2 - 2x - 1$
a) $P(3) = 2$
$27 + 9a - 6 - 1 = 2$
$9a = -18$
$a = -2$
b) 1 é raiz. $\Rightarrow P(1) = 0$
$1 + a - 2 - 1 = 0$
$a = 2$

21.05) $P(x) = ax + b$
$P(0) = 3 \Rightarrow b = 3$
$P(-1) = 2 \Rightarrow -a + b = 2 \Rightarrow a = 1$
$P(x) = x + 3$

21.06) $P(x) = (4x^3 - 2x^2 - 2x - 1)^3$
Soma dos coeficientes
$P(1) = (4 - 2 - 2 - 1)^3 = (-1)^3 = -1$

21.07) $P(x) + x \cdot P(2 - x) = x^2 + 3$
$x = 0$
$\Rightarrow P(0) + 0 \cdot P(2 - 0) = 0 + 3$
$P(0) = 3$

$x = 1$
$P(1) + 1 \cdot P(2 - 1) = 1 + 3$
$2P(1) = 4$
$P(1) = 2$

$x = 2$
$P(2) + 2 \cdot P(2 - 2) = 4 + 3$
$P(2) + 2 \cdot P(0) = 7$
$P(2) + 6 = 7$
$P(2) = 1$

21.08) E
$P(x) = x^3 + ax^2 + bx + c$
$P(1) = 0 \Rightarrow 1 + a + b + c = 0$
$a + b + c = -1$
$P(-x) + P(x) = 0$

$-\cancel{x^3} + ax^2 - \cancel{bx} + c + \cancel{x^3} + ax^2 + \cancel{bx} + c = 0$
$2ax^2 + 2c = 0$

$\Rightarrow \begin{cases} 2a = 0 \Rightarrow a = 0 \\ 2c = 0 \Rightarrow c = 0 \end{cases}$

Como $a + b + c = -1$
$b = -1$
Assim, $P(x) = x^3 - x$
$P(2) = 8 - 2 = 6$

21.09) C
P(x) tem grau 5.
Exemplo: $P(x) = x^5$
$[P(x)]^3 + [P(x)]^2 + 2P(x)$
$= (x^5)^3 + (x^5)^2 + 2 \cdot x^5$
$= x^{15} + x^{10} + 2x^5$
Grau 15
Observação: O grau é 15 independentemente do polinômio P(x) tomado como exemplo.

21.10) Considere os exemplos abaixo.

f	g	h	f . (g+h)	Grau **n**
x^3	x^3	$-x^3$	0	Não existe.
x^3	$x^3 + 1$	$-x^3$	x^3	3
x^3	$x^3 + x$	$-x^3$	x^4	4
x^3	$x^3 + x^2$	$-x^3$	x^5	5
x^3	x^3	x^3	$2x^6$	6

Assim, $3 \leq n \leq 6$.

21.11) $P(x) = (a^2 - 4) \cdot x^5 + (a + 2) \cdot x^3 + (a - 2) \cdot x + 7$
Basta identificar os valores de **a** que anulam os coeficientes.
$a^2 - 4 = 0 \Rightarrow a = \pm 2$
$a + 2 = 0 \Rightarrow a = -2$
$a - 2 = 0 \Rightarrow a = 2$

Conclusão
Se $a = 2 \Rightarrow P(x) = 4x^3 + 7 \Rightarrow$ grau = 3.
Se $a = -2 \Rightarrow P(x) = -2x + 7 \Rightarrow$ grau = 1.
Se $a \neq \pm 2 \Rightarrow P(x)$ tem grau = 5.

21.12) **Falso**.
$k = 4 \Rightarrow P(x) = (4 - 4)x^2 + 4^2x + 2$
$P(x) = 16x + 2$
Grau = 1
Falso.
É necessário que $k - 4 \neq 0 \Rightarrow k \neq 4$.
Verdadeiro.
$P(x) = (k - 4) \cdot x^2 + k^2x + 2 = 0$
Para $x = -k$
$P(-k) = (k - 4) \cdot K^2 + k^2 \cdot (-K) + 2 = 0$
$P(-k) = K^3 - 4k^2 - k^3 + 2$
$P(-K) = -4k^2 + 2$
Falso.
-1 é raiz $\Rightarrow P(-1) = 0$
$(k - 4) \cdot (-1)^2 + k^2 \cdot (-1) + 2 = 0$
$k - 4 - k^2 + 2 = 0$
$-k^2 + k - 2 = 0 \qquad (-1)$
$k^2 - k + 2 = 0$
$\Delta = 1 - 4 \cdot 1 \cdot 2 = -7$
$S = \varnothing$.
Verdadeiro.
$k = 5 \Rightarrow P(x) = (5 - 4) \cdot x^2 + 5^2 \cdot x + 2$
$X^2 + 25X + 2 = 0$
$\Delta = (25)^2 - 4 \cdot 1 \cdot 2$
$\Delta = 617 > 0$
P(x) admite raízes reais.

21.13) $P(x) = (2a + b) \cdot x^2 + (b - c) \cdot x + a + c - 4 \equiv 0$

$$\begin{cases} 2a + b = 0 & (I) \\ b - c = 0 \Rightarrow b = c & (II) \\ a + c - 4 = 0 & (III) \end{cases}$$

Substituindo II em I e resolvendo com III, temos:

$$\begin{cases} 2a + c = 0 \\ a + c = 4 \quad .(-1) \end{cases}$$

$$\begin{cases} 2a + \cancel{c} = 0 \\ -a - \cancel{c} = -4 \end{cases} \oplus$$

$a = -4$
$c = 8$
$b = 8$

21.14) $P(x) = ax^5 + (b - a + 1) \cdot x^3 + (c + b + a) \cdot x \equiv 0$
$a = 0$
$b - a + 1 = 0 \Rightarrow b = -1$
$c + b + a = 0 \Rightarrow c = 1$

21.15) $P(x) = ax^4 + bx^3 + c$
$Q(x) = ax^3 - bx + c$
$P(0) = 0 \Rightarrow c = 0$

$$\begin{matrix} P(1) = 0 \Rightarrow \\ Q(1) = 6 \Rightarrow \end{matrix} \begin{cases} a + b = 0 \\ a - b = 6 \end{cases} \oplus$$

$2a = 6$
$a = 3$
$b = -3$

21.16) $P(x) = ax^2 + bx + c$
$P(1) = 4 \Rightarrow a + b + c = 4$
$P(0) + P(-1) = 3 \Rightarrow c + a - b + c = 3$
$P(-2) = -5 \Rightarrow 4a - 2b + c = -5$

$$\begin{cases} a + b + c = 4 \Rightarrow c = 4 - a - b & (I) \\ a - b + 2c = 3 & (II) \\ 4a - 2b + c = -5 & (III) \end{cases}$$

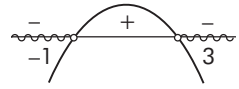
Substituindo I em II e III, temos:

$$\begin{cases} a - 2 + 2 \cdot (4 - a - b) = 3 \\ 4a - 2b + 4 - a - b = -5 \end{cases}$$

$$\begin{cases} -a - 2b = -3 \\ 3a - 3b = -9 \quad \div (3) \end{cases}$$

$$\begin{cases} -\cancel{a} - 2b = -3 \\ \cancel{a} - b = -3 \end{cases} \oplus$$

$-3b = -6$
$b = 2$; $a = -1$; $c = 3$
$P(x) = -x^2 + 2x + 3$; $P(x) < 0$
$x' = -1$; $x'' = 3$



$P(x) < 0$ para $x < -1$ ou $x > 3$

21.17) $f(x) = (a - 1) \cdot x^2 + bx + c$
$g(x) = 2ax^2 + 2bx - c$
$f(x) \equiv g(x)$
$\Rightarrow a - 1 = 2a \Rightarrow a = -1$
$b = 2b \Rightarrow b = 0$
$c = -c \Rightarrow c = 0$

21.18) $P_1(x) = (m + n + p) \cdot x^4 - (p + 1) \cdot x^3 + mx^2 + (n - p) \cdot x + n$
$P_2(x) = 2mx^3 + (2p + 7) \cdot x^2 + 5mx + 2m$
$P_1(x) \equiv P_2(x)$
$\Rightarrow n = 2m$
$n - p = 5m \Rightarrow 2m - p = 5m \Rightarrow p = -3m$
$m = 2p + 7 \Rightarrow m = -6m + 7 \Rightarrow m = 1$
$p = -3$
$n = 2$
$m + n + p = 0$
Isto ocorre, pois $m = 1$; $p = -3$ e $n = 2$.

21.19) $P(x) = 2x^2 + 3x + 1$
$Q(x) = (ax + b) \cdot (x + 1)$
$= ax^2 + ax + bx + b$
$= ax^2 + (a + b) \cdot x + b$
Como $P(x) \equiv Q(x)$:
$a = 2$
$a + b = 3 \Rightarrow b = 1$

21.20) C
$P(x) = x^2 + 2x - 8$
$Q(x) = m \cdot (x - 2) \cdot (x - 3) + n \cdot (x - 1) \cdot (x - 2)$
$= m \cdot (x^2 - 3x - 2x + 6) + n \cdot (x^2 - 2x - x + 2)$
$= mx^2 - 5mx + 6m + nx^2 - 3nx + 2n$
$= (m + n) \cdot x^2 + (-5m - 3n) \cdot x + 6m + 2n$

$P(x) \equiv Q(x)$

$\begin{cases} m + n = 1 \\ -5m - 3n = 2 \end{cases} \cdot (3)$

$\begin{cases} 3m + 3n = 3 \\ -5m - 3n = 2 \end{cases} \oplus$

$-2m = 5$

$m = -\frac{5}{2}$

$n = \frac{7}{2}$

Observe que a terceira igualdade fica satisfeita

$6m + 2n = -8$ para os valores obtidos de **m** e **n**.

21.21) $P_2(x) = 10x^2 + 158x + 29$

$P_1(x) = ax^2 + (b + c) \cdot x - 2a - 3x^2 + 3cx + 3b + 1$

$= (a - 3) \cdot x^2 + (b + 4c) \cdot x - 2a + 3b + 1$

$P_1(x) \equiv P_2(x)$

$a - 3 = 10 \Rightarrow a = 13$

$-2a + 3b + 1 = 29 \Rightarrow -26 + 3b + 1 = 29$

$\Rightarrow b = 18$

$b + 4c = 158 \Rightarrow 18 + 4c = 158 \Rightarrow c = 35$

Logo, $a + b + c = 13 + 18 + 35 = 66$.

21.22) $P(x) = 2x^3 + 3x^2 - 2x - 3$

$Q(x) = (ax^2 + bx + c) \cdot (x + 1)$

$= ax^3 + ax^2 + bx^2 + bx + cx + c$

$= ax^3 + (a + b) \cdot x^2 + (b + c) \cdot x + c$

$P(x) \equiv Q(x)$

$a = 2$

$a + b = 3 \Rightarrow b = 1$

$b + c = -2 \Rightarrow c = -3$

21.23) D

$P(x) = px^2 + qx - 4$

$Q(x) = x^2 + px + q$

$P(x + 1) = p(x + 1)^2 + q(x + 1) - 4$

$= p(x^2 + 2x + 1) + qx + q - 4$

$= px^2 + (2p + q) \cdot x + p + q - 4$

$Q(2x) = (2x^2) + p \cdot (2x) + q$

$= 4x^2 + 2px + q$

Como $P(x + 1) \equiv Q(2x)$:

$p = 4$

$2p + q = 2p \Rightarrow q = 0$

$p + q - 4 = q \Rightarrow p = 4$

21.24) C

$\frac{1 + x}{x - x^2} = \frac{A}{x} + \frac{B}{1 - x}$

$\frac{1 + x}{x - x^2} = \frac{A(1 - x) + Bx}{x \cdot (1 - x)}$

$1 + x = A - Ax + Bx$

$x + 1 = (B - A) \cdot x + A$

$\Rightarrow A = 1$

$B - A = 1 \Rightarrow B = 2$

21.25) $\frac{1}{x^3 - 1} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + x + 1}$

$\frac{1}{x^3 - 1} = \frac{Ax^2 + Ax + A + (Bx + C) \cdot (x - 1)}{(x - 1) \cdot (x^2 + x + 1)}$

$\frac{1}{x^3 - 1} = \frac{Ax^2 + Ax + A + Bx^2 - Bx + Cx - C}{x^3 - 1}$

$1 = (A + B) \cdot x^2 + (A - B + C) \cdot x + A - C$

$\Rightarrow \begin{cases} A + B = 0 \Rightarrow B = -A \\ A - B + C = 0 \Rightarrow A + A + C = 0 \Rightarrow 2A + C = 0 \quad (I) \\ A - C = 1 \quad (II) \end{cases}$

De I e II, temos:

$\begin{cases} 2A + C = 0 \\ A - C = 1 \end{cases} \oplus$

$3A = 1$

$\Rightarrow A = \frac{1}{3}$

$B = -\frac{1}{3}$

$C = -\frac{2}{3}$

21.26) C

$\frac{2x - 4}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1}$

$\frac{2x - 4}{x^2 - 1} = \frac{Ax - A + Bx + B}{(x + 1) \cdot (x - 1)}$

$\frac{2x - 4}{x^2 - 1} = \frac{(A + B) \cdot x - A + B}{x^2 - 1}$

$2x - 4 = (A + B) \cdot x - A + B$

$\Rightarrow \begin{cases} A + B = 2 \\ -A + B = -4 \end{cases} \oplus$

$2B = -2$

$B = -1; A = 3$

21.27) E

$\frac{8}{x^3 - 4x} = \frac{A}{x} + \frac{B}{x - 2} + \frac{C}{x + 2}$

$\frac{8}{x^3 - 4x} \pm \frac{A(x^2 - 4) + B(x^2 + 2x) + C(x^2 - 2x)}{x \cdot (x - 2) \cdot (x + 2)}$

$\frac{8}{x^3 - 4x} = \frac{Ax^2 - 4A + Bx^2 + 2Bx + Cx^2 - 2Cx}{x \cdot (x^2 - 4)}$

$8 = (A + B + C) \cdot x^2 + (2B - 2C) \cdot x - 4A$

$\Rightarrow -4A = 8 \Rightarrow A = -2$

$A + B + C = 0 \Rightarrow B + C = 2 \quad (I)$

$2B - 2C = 0 \div (2) \Rightarrow B - C = 0 \quad (II)$

De I e II, obtemos:

$$\begin{cases} B + C = 2 \\ B - C = 0 \end{cases} \oplus$$

$2B = 2$
$B = 1; C = 1$

Aula 22

22.01) D
$P(x) \equiv D(x) \cdot Q(x) + R(x)$
$ax^3 + bx^2 + cx + d = (x^2 + 1) \cdot (2x - 1) + (-x + 1)$
$= 2x^3 - x^2 + 2x - 1 - x + 1$
$= 2x^3 - x^2 + x$
$\Rightarrow a = 2; b = -1; c = 1; d = 0$
$P(x) = 2x^3 - x^2 + x$

22.02) $P(x) \equiv D(x) \cdot Q(x) + R(x)$
$= (2x + 3) \cdot (x - 1) + 6$
$= 2x^2 - 2x + 3x - 3 + 6$
$= 2x^2 + x + 3$

22.03) $P(x) \equiv D(x) \cdot Q(x) + R(x)$
$= (3x - 4) \cdot (x^2 - 5x + 1) + 6$
$= 3x^3 - 15x^2 + 3x - 4x^2 + 20x - 4 + 6$
$= 3x^3 - 19x^2 + 23x + 2$

22.04) $A(x) + B(x) \cdot C(x)$
$= x^2 - 1 + (2x^2 + 2x) \cdot 3x$
$= x^2 - 1 + 6x^3 + 6x^2$
$= 6x^3 + 7x^2 - 1$

22.05) E
$f(x) \equiv D(x) \cdot Q(x) + R(x)$
$= (x - 3) \cdot (x - 4) - 7$
$= x^2 - 4x - 3x + 12 - 7$
$= x^2 - 7x + 5$

22.06) E
$P(x) \equiv (2x - 1) \cdot (x^2 - x) + m$
$= 2x^3 - 2x^2 - x^2 + x + m$
$= 2x^3 - 3x^2 + x + m$
$P(-1) = 0 \Rightarrow -2 - 3 - 1 + m = 0$
$m = 6$

22.07) a) **Verdadeira**.
b) **Verdadeira**.
c) **Falsa**.
Se, por exemplo, $n = 3$ e
$P(x) = x^3 + 2$ e
$Q(x) = -x^3 + 7$, então
$P(x) + Q(x) = 9$ não tem grau 3.
d) **Verdadeira**.
e) **Falsa**.
Se $gr(P) = 5$ e $gr(Q) = 3$, então $gr(P + Q) = 5$.
f) **Falsa**.
Se $P(x) = x^5$ e $Q(x) = x^3$, então $P(x) \cdot Q(x) = x^8$.

22.08) a) $3x^5 - x^4 + 2x^3 + 4x - 3 = (x^3 - 2x + 1) \cdot (ax^2 + bx + c) + dx^2 + ex + f$
$= ax^5 + bx^4 + cx^3 - 2ax^3 - 2bx^2 - 2cx + ax^2 + bx + c + dx^2 + ex + f$
$= ax^5 + bx^4 + (c - 2a) \cdot x^3 + (a - 2b + d) \cdot x^2 + (-2c + b + e) \cdot x + c + f$
$\Rightarrow a = 3$
$b = -1$
$c - 2a = 2 \Rightarrow c = 8$
$a - 2b + d = 0 \Rightarrow d = -5$
$-2c + b + e = 4 \Rightarrow e = 21$
$c + f = -3 \Rightarrow f = -11$
Logo, $Q(x) = 3x^2 - x + 8$; $R(x) = -5x^2 + 21x - 11$.
b) $x^4 - 2x + 13 = (x^2 + x + 1) \cdot (ax^2 + bx + c) + dx + e$
$= ax^4 + bx^3 + cx^2 + ax^3 + bx^2 + cx + ax^2 + bx + c + dx + e$
$= ax^4 + (a + b) \cdot x^3 + (a + b + c) \cdot x^2 + (b + c + d) \cdot x + c + e$
$\Rightarrow a = 1$
$a + b = 0 \Rightarrow b = -1$
$a + b + c = 0 \Rightarrow c = 0$
$b + c + d = -2 \Rightarrow d = -1$
$c + e = 13 \Rightarrow e = 13$
Logo, $Q(x) = x^2 - x$; $R(x) = -x + 13$.
c) $2x^5 - 3x + 12 = (x^2 + 1) \cdot (ax^3 + bx^2 + cx + d) + ex + f$
$= ax^5 + bx^4 + cx^3 + dx^2 + ax^3 + bx^2 + cx + d + ex + f$
$= ax^5 + bx^4 + (a + c) \cdot x^3 + (b + d) \cdot x^2 + (c + e) \cdot x + d + f$
$\Rightarrow a = 2$
$b = 0$
$a + c = 0 \Rightarrow c = -2$
$b + d = 0 \Rightarrow d = 0$
$c + e = -3 \Rightarrow e = -1$
$d + f = 12 \Rightarrow f = 12$
Logo, $Q(x) = 2x^3 - 2x$; $R(x) = -x + 12$

22.09) a) $4x^2 - 6x + 2 = (x + 1) \cdot (ax + b) + c$
$= ax^2 + bx + ax + b + c$
$= ax^2 + (a + b) \cdot x + b + c$
$\Rightarrow a = 4$
$a + b = -6 \Rightarrow b = -10$
$b + c = 2 \Rightarrow c = 12$
Logo, $Q(x) = 4x - 10$; $R(x) = 12$
b) $8x^3 + 12x^2 + 6x + 1 = (2x + 2) \cdot (ax^2 + bx + c) + d$
$= 2ax^3 + 2bx^2 + 2cx + 2ax^2 + 2bx + 2c + d$
$= 2ax^3 + (2a + 2b) \cdot x^2 + (2b + 2c) \cdot x + 2c + d$
$\Rightarrow 2a = 8 \Rightarrow a = 4$
$2a + 2b = 12 \Rightarrow b = 2$
$2b + 2c = 6 \Rightarrow c = 1$
$2c + d = 1 \Rightarrow d = -1$
Logo, $Q(x) = 4x^2 + 2x + 1$; $R(x) = -1$.
c) $x^4 + 6x^2 + 8 = (x^2 - 1) \cdot (ax^2 + bx + c) + dx + e$
$= ax^4 + bx^3 + cx^2 - ax^2 - bx - c + dx + e$
$= ax^4 + bx^3 + (c - a) \cdot x^2 + (d - b) \cdot x - c + e$
$\Rightarrow a = 1$
$b = 0$

$c - a = 6 \Rightarrow c = 7$
$d - b = 0 \Rightarrow d = 0$
$-c + e = 8 \Rightarrow e = 15$
Logo, $Q(x) = x^2 + 7$; $R(x) = 15$.

22.10) $x^3 + px^2 + qx - 1 = (x^2 - x + 2) \cdot (ax + b) - 7$
$= ax^3 + bx^2 - ax^2 - bx + 2ax + 2b - 7$
$= ax^3 + (b - a) \cdot x^2 + (2a - b) \cdot x + 2b - 7$
$\Rightarrow a = 1$
$2b - 7 = -1 \Rightarrow b = 3$
$b - a = p \Rightarrow p = 2$
$2a - b = q \Rightarrow q = -1$

22.11) $4x^3 + 5x^2 - 3x + 6 = (x^2 + x - 1) \cdot (ax + b) +$
$+ cx + d$
$= ax^3 + bx^2 + ax^2 + bx - ax - b + cx + d$
$= ax^3 + (a + b) \cdot x^2 + (-a + b + c) \cdot x + d - b$
$\Rightarrow a = 4$
$a + b = 5 \Rightarrow b = 1$
$-a + b + c = -3 \Rightarrow c = 0$
$d - b = 6 \Rightarrow d = 7$
Logo, $Q(x) = 4x + 1$; $R(x) = 7$.

22.12) $x^3 + 1 = (x^2 - x + 1) \cdot (ax + b) + cx + d$
$= ax^3 + bx^2 - ax^2 - bx + ax + b + cx + d$
$= ax^3 + (b - a) \cdot x^2 + (a - b + c) \cdot x + b + d$
$\Rightarrow a = 1$
$b - a = 0 \Rightarrow b = 1$
$a - b + c = 0 \Rightarrow c = 0$
$b + d = 1 \Rightarrow d = 0$
Logo, $Q(x) = x + 1$; $R(x) = 0$.

22.13) Divisível $\Rightarrow R(x) = 0$
$x^4 + ax^3 - 8x^2 + bx + 15 = (x^2 + 2x + 3).$
$. (px^2 + qx + r) + 0$
$= px^4 + qx^3 + rx^2 + 2px^3 + 2qx^2 + 2rx + 3px^2$
$+ 3qx + 3r$
$= px^4 + (q + 2p) \cdot x^3 + (3p + 2q + r) \cdot x^2 +$
$+ (3q + 2r) \cdot x + 3r$
$\Rightarrow p = 1$
$3r = 15 \Rightarrow r = 5$
$3p + 2q + r = -8 \Rightarrow q = -8$
$a = q + 2p \Rightarrow a = -6$
$b = 3q + 2r \Rightarrow b = -14$

22.14) E
Disivísvel $\Rightarrow R(x) = 0$
$2x^4 - 3x^3 + ax^2 + bx + c = (x^3 - 7x + 6) \cdot (px + q) +$
$+ 0$
$= px^4 + qx^3 - 7px^2 - 7qx + 6px + 6q$
$= px^4 + qx^3 - 7px^2 + (6p - 7q) \cdot x + 6q$
$\Rightarrow p = 2$
$q = -3$
$a = -7p \Rightarrow a = -14$
$b = 6p - 7q \Rightarrow b = 33$
$c = 6q \Rightarrow c = -18$

22.15) Divisão exata $\Rightarrow R(x) = 0$
$x^4 + mx^2 + n = (x^2 - x + 1) \cdot (ax^2 + bx + c) + 0$
$= ax^4 + bx^3 + cx^2 - ax^3 - bx^2 - cx + ax^2 + bx + c$
$= ax^4 + (b - a) \cdot x^3 + (a - b + c) \cdot x^2 +$
$+ (b - c) \cdot x + c$
$\Rightarrow a = 1$
$b - a = 0 \Rightarrow b = 1$
$b - c = 0 \Rightarrow c = 1$
$m = a - b + c \Rightarrow m = 1$
$n = c \Rightarrow n = 1$

22.16) $A(x) \cdot B(x) - C(x)$
$= x \cdot (x^3 + x^2 + 1) - (x^4 - x^3 + x)$
$= \cancel{x^4} + x^3 + \cancel{x} - \cancel{x^4} + x^3 - \cancel{x} = 2x^3$
Grau = 3.

22.17) A
Exemplo
$f(x) = x^4$
$g(x) = x^6$
$h(x) = x^3$
$(f + g) \cdot h = (x^4 + x^6) \cdot x^3 = x^7 + x^9$
Grau = 9

22.18) Exemplo

x^{10} | x^3
$Q(x)$: grau $= 10 - 3 = 7$

$R(x)$: grau máximo $= 3 - 1 = 2$

a) 7
b) 2

22.19) A

x^m | $D(x)$
x^{m-4}

$gr(P) = gr(D) + gr(Q)$
$m = gr(D) + m - 4$
$gr(D) = 4$
$gr(R) \leq gr(D) - 1$
$gr(R) \leq 3$

22.20) A
Exemplo
$p(x) = x^4$
$q(x) = x^5$
$p(x) + q(x) = x^4 + x^5$
Grau = 5

22.21) 29
01. **Verdadeira**.
$gr(Q) = gr(P) - gr(D) = 10 - 1 = 9$
02. **Falsa**. O resto pode ter grau, no máximo, igual a 17.

04. **Verdadeira**. Teoria.
08. **Verdadeira**.
gr(Q) = gr(P) – gr(Q) = n + 2 – 2 = n
16. **Verdadeira**. Teoria.

Aula 23

23.01) a)

$x^3 - x^2 + 1 \;\big|\; x + 1$

$-x^3 - x^2 \qquad\qquad x^2 - 2x + 2$

$-2x^2 + 1$

$2x^2 + 2x$

$2x + 1$

$-2x - 2$

-1

$Q(x) = x^2 - 2x + 2$
$R(x) = -1$

b)

$2x^2 - 5x + 1 \;\big|\; x - 2$

$-2x^2 + 4x \qquad\qquad 2x - 1$

$-x + 1$

$x - 2$

-1

$Q(x) = 2x - 1$
$R(x) = -1$

c)

$2x^3 - 5x^2 + 3x + 1 \;\big|\; 2x + 1$

$-2x^3 - x^2 \qquad\qquad x^2 - 3x + 3$

$-6x^2 + 3x + 1$

$6x^2 + 3x$

$6x + 1$

$-6x - 3$

-2

$Q(x) = x^2 - 3x + 3$
$R(x) = -2$

23.02) a)

$x^4 + 4x^3 + 6x^2 + 4x + 1 \;\big|\; x + 1$

$-x^4 - x^3 \qquad\qquad x^3 + 3x^2 + 3x + 1$

$3x^3 + 6x^2 + 4x + 1$

$-3x^3 - 3x^2$

$3x^2 + 4x + 1$

$-3x^2 - 3x$

$x + 1$

$-x - 1$

0

$Q(x) = x^3 + 3x^2 + 3x + 1$
$R(x) = 0$

b)

$x^3 + 8x^2 + 21x + 18 \;\big|\; x + 3$

$-x^3 - 3x^2 \qquad\qquad x^2 + 5x + 6$

$5x^2 + 21x + 18$

$-5x^2 - 15x$

$6x + 18$

$-6x - 18$

0

$Q(x) = x^2 + 5x + 6$
$R(x) = 0$

c)

$x^3 - 8 \;\big|\; x^2 + 2x + 4$

$-x^3 - 2x^2 - 4x \qquad\qquad x - 2$

$-2x^2 - 4x - 8$

$2x^2 + 4x + 8$

0

$Q(x) = x - 2$
$R(x) = 0$

23.03) a)

$x^2 + 5x + 1 \;\big|\; 2x^2 + 4x - 3$

$-x^2 - 2x + 3/2 \qquad\qquad 1/2$

$3x + 5/2$

$Q(x) = \frac{1}{2}$

$R(x) = 3x + \frac{5}{2}$

b) $x^4 + 2x^3 + x^2 + 4x - 2 \;\big|\; x^2 + 2$

$-x^4 - 2x^2 \qquad\qquad x^2 + 2x - 1$

$2x^3 - x^2 + 4x - 2$

$-2x^3 - 4x$

$-x^2 - 2$

$x^2 + 2$

0

$Q(x) = x^2 + 2x - 1$

$R(x) = 0$

c) $5x + 1 \;\big|\; x^3 + 5$

Impossível efetuar, já que gr(P) < gr(D).

$Q(x) = 0$

$R(x) = 5x + 1$

d) $3x^3 + 6x^2 + 9 \;\big|\; 3x^2 + 1$

$-3x^3 - x \qquad\qquad x + 2$

$6x^2 - x + 9$

$-6x^2 - 2$

$-x + 7$

$Q(x) = x + 2$

$R(x) = -x + 7$

e) $x^3 + x^2 + x + 1 \;\big|\; 2x^2 + 3$

$-x^3 - \frac{3x}{2} \qquad\qquad \frac{1}{2}x + \frac{1}{2}$

$x^2 - \frac{x}{2} + 1$

$-x^2 - \frac{3}{2}$

$\frac{-x}{2} - \frac{1}{2}$

$Q(x) = \frac{1}{2}x + \frac{1}{2}$

$R(x) = -\frac{1}{2}x - \frac{1}{2}$

23.04) a) $3x^4 - 2x^3 - x^2 + 5x + 1 \;\big|\; x^2 - 3x + 1$

$-3x^4 + 9x^3 - 3x^2 \qquad\qquad 3x^2 + 7x + 17$

$7x^3 - 4x^2 + 5x + 1$

$-7x^3 + 21x^2 - 7x$

$17x^2 - 2x + 1$

$-17x^2 + 51x - 17$

$49x - 16$

$Q(x) = 3x^2 + 7x + 17$

$R(x) = 49x - 16$

b) $5x^3 + 1 \;\big|\; x + 3$

$-5x^3 - 15x^2 \qquad\qquad 5x^2 - 15x + 45$

$-15x^2 + 1$

$15x^2 + 45x$

$45x + 1$

$-45x - 135$

-134

$Q(x) = 5x^2 - 15x + 45$

$R(x) = -134$

c) $8x^3 + 4x^2 - x - 1 \;\big|\; -x$

$-8x^3 \qquad\qquad -8x^2 - 4x + 1$

$4x^2 - x - 1$

$-4x^2$

$-x - 1$

x

-1

$Q(x) = -8x^2 - 4x + 1$

$R(x) = -1$

d) $x^3 + 2x + 4$ | $\frac{x}{2} + 1$

$-x^3 - 2x^2$ → $2x^2 - 4x + 12$

$-2x^2 + 2x + 4$

$2x^2 + 4x$

$6x + 4$

$-6x - 12$

-8

$Q(x) = 2x^2 - 4x + 12$
$R(x) = -8$

e) $\frac{4}{3}x^3 + x^2$ | $x - 1$

$-\frac{4}{3}x^3 + \frac{4}{3}x^2$ → $\frac{4}{3}x^2 + \frac{7}{3}x + \frac{7}{3}$

$\frac{7}{3}x^2$

$-\frac{7}{3}x^2 + \frac{7}{3}x$

$\frac{7}{3}x$

$-\frac{7}{3}x + \frac{7}{3}$

$\frac{7}{3}$

$Q(x) = \frac{4x^2}{3} + \frac{7}{3}x + \frac{7}{3}$

$R(x) = \frac{7}{3}$

23.05) D

$2x^5 - x^4 + x^2$ | $2x + 3$

$-2x^5 - 3x^4$ → $x^4 - 2x^3 + 3x^2 - 4x + 6$

$-4x^4 + x^2$

$4x^4 + 6x^3$

$6x^3 + x^2$

$-6x^3 - 9x^2$

$-8x^2$

$8x^2 + 12x$

$12x$

$-12x - 18$

-18

$Q(x) = x^4 - 2x^3 + 3x^2 - 4x + 6$
$R(x) = -18$

23.06) D

$g(x) = (x - 2) \cdot (x - 3) = x^2 - 5x + 6$

$x^3 - x^2 + x - 1$ | $x^2 - 5x + 6$

$-x^3 + 5x^2 - 6x$ → $x + 4$

$4x^2 - 5x - 1$

$-4x^2 + 20x - 24$

$15x - 25$

$Q(x) = x + 4$
$R(x) = 15x - 25$

23.07) B

$x^3 + px + q$ | $x^2 + x + 1$

$-x^3 - x^2 - x$ → $x - 1$

$-x^2 + (p-1)x + q$

$x^2 + x + 1$

$px + q + 1$

$Q(x) = x - 1$

23.08) A

$f + g = 2x^2 - 6x + 1 + x^3 + 2$
$= x^3 + 2x^2 - 6x + 3$

$x^3 + 2x^2 - 6x + 3$ | $x - 1$
$-x^3 + x^2$ | $x^2 + 3x - 3$
$3x^2 - 6x + 3$
$-3x^2 + 3x$
$-3x + 3$
$3x - 3$
0

$Q(x) = x^2 + 3x - 3$

23.09) D

$2x^3 - 21x^2 + 5x - 1$ | $x + 1$
$-2x^3 - 2x^2$ | $2x^2 - 23x + 28$
$-23x^2 + 5x - 1$
$23x^2 + 23x$
$28x - 1$
$-28x - 28$
-29

$Q(x) = 2x^2 - 23x + 28$

23.10) D

$x^3 - 2x^2 + 4$ | $x^2 - 4$
$-x^3 + 4x$ | $x - 2$
$-2x^2 + 4x + 4$
$2x^2 - 8$
$4x - 4$

$R(x) = 4x - 4$

23.11) D

$x^3 - 2x^2 + x - 1$ | $x^2 - x + 1$
$-x^3 + x^2 - x$ | $x - 1$
$-x^2 - 1$
$x^2 - x + 1$
$-x$

$R(x) = -x \Rightarrow R(1) = -1$

23.12) A

$x^4 - 10x^3 + 24x^2 + 10x - 24$ | $x^2 - 6x + 5$
$-x^4 + 6x^3 - 5x^2$ | $x^2 - 4x - 5$
$-4x^3 + 19x^2 + 10x - 24$
$4x^3 - 24x^2 + 20x$
$-5x^2 + 30x - 24$
$5x^2 - 30x + 25$
1

$Q(x) = x^2 - 4x - 5 = 0$
$x' = 5; x'' = -1$

23.13) B

Divisível $\Rightarrow$ resto = 0

$x^4 - 4x^3 - 10x^2 + ax + b$ | $x^2 - x + 5$
$-x^4 + x^3 - 5x^2$ | $x^2 - 3x - 18$
$-3x^3 - 15x^2 + ax + b$
$3x^3 - 3x^2 + 15x$
$-18x^2 + (a + 15)x + b$
$18x^2 - 18x + 90$
$(a - 3)x + b + 90$

$R(x) = (a - 3) \cdot x + b + 90 \equiv 0$
$\Rightarrow a - 3 = 0$
$a = 3$
$b + 90 = 0$
$b = -90$
Logo, $a + b = 3 - 90 = -87$.

23.14) E

Divisível $\Rightarrow$ resto = 0

$x^3 + 2x^2 + mx + n$ | $x^2 + x + 1$
$-x^3 - x^2 - x$ | $x + 1$
$x^2 + (m - 1)x + n$
$-x^2 - x - 1$
$(m - 2)x + n - 1$

Resto $= (m - 2) \cdot x + n - 1 \equiv 0$
$\Rightarrow m - 2 = 0$
$m = 2$
$n - 1 = 0$
$n = 1$
Logo, $m + n = 2 + 1 = 3$.

23.15) C

Divisível $\Rightarrow$ resto = 0

$x^4 - 12x^3 + 47x^2 + mx + n \;|\; x^2 - 7x + 6$

$-x^4 + 7x^3 - 6x^2 \qquad x^2 - 5x + 6$

$-5x^3 + 41x^2 + mx + n$

$5x^3 - 35x^2 + 30x$

$+6x^2 + (m + 30)x + n$

$-6x^2 + 42x - 36$

$(m + 72)x + n - 36$

$R(x) = (m + 72) \cdot x + n - 36 \equiv 0$

$m + 72 = 0$

$m = -72$

$n - 36 = 0$

$n = 36$

Logo, $m + n = -72 + 36 = -36$

23.16) B

Divisível $\Rightarrow$ resto = 0

$x^3 + 2x^2 + px + q \;|\; x^2 + x + 1$

$-x^3 - x^2 - x \qquad x + 1$

$x^2 + (p - 1)x + q$

$-x^2 - x - 1$

$(p - 2)x + q - 1$

$R(x) = (p - 2) \cdot x + q - 1 \equiv 0$

$p - 2 = 0$

$p = 2$

$q - 1 = 0$

$q = 1$

Logo, $p + q = 2 + 1 = 3$.

23.17) a) $2x^3 + Ax + 3B \;|\; x^2 - 3x + 9$

$-2x^3 + 6x^2 - 18x \qquad 2x + 6$

$6x^2 + (A - 18)x + 3B$

$-6x^2 + 18x - 54$

$Ax + 3B - 54$

b) Divisão exata $\Rightarrow$ resto = 0

$R(x) = Ax + 3B - 54 = 0$

$A = 0$

$3B - 54 = 0$

$B = 18$

23.18) D

Divisível $\Rightarrow$ resto = 0

$x^3 + px + q \;|\; x^2 + 2x + 5$

$-x^3 - 2x^2 - 5x \qquad x - 2$

$-2x^2 + (p - 5)x + q$

$2x^2 + 4x + 10$

$(p - 1)x + q + 10$

$R(x) = (p - 1) \cdot x + q + 10 \equiv 0$

$p - 1 = 0$

$p = 1$

$q + 10 = 0$

$q = -10$

23.19) Divisível $\Rightarrow$ resto = 0

$2x^3 - ax + b \;|\; x^2 - 2$

$-2x^3 + 4x \qquad 2x$

$(4 - a)x + b$

$R(x) = (4 - a) \cdot x + b \equiv 0$

$\Rightarrow 4 - a = 0$

$a = 4$

$b = 0$

23.20) I. **Falsa**.

Exemplo

$x^n + x^n = 2x^n$

Grau **n**.

II. **Falsa**.

Exemplo:

$x^n \cdot x^n = x^{2n}$

Grau 2n.

III. **Falsa**.

gr(resto) < gr(divisor)

gr(resto) < n – 3

23.21) Divisível $\Rightarrow$ resto = 0

$2x^4 - 3x^3 - ax^2 + bx - c \;|\; x^3 - 7x + 6$

$-2x^4 + 14x^2 - 12x \qquad 2x - 3$

$-3x^3 + (14 - a)x^2 + (b - 12)x - c$

$3x^3 - 21x + 18$

$(14 - a)x^2 + (b - 33)x + 18 - c$

Resto = $(14 - a) \cdot x^2 + (b - 33) \cdot x + 18 - c \equiv 0$

$\Rightarrow a = 14;\ b = 33;\ c = 18$

Logo, $a + b + c = 65$

Aula 24

24.01) E

Resto = P(–1) = 1 + 2 + 1 + 1 + 1 = 6

24.02) D

$$\text{Resto} = P\left(\frac{1}{2}\right) = 2 \cdot \frac{1}{16} - \frac{3}{2} + 1 =$$

$$= \frac{1}{8} - \frac{3}{2} + 1 \quad = \frac{1 - 12 + 8}{8} = \frac{-3}{8}$$

24.03) D

Resto = P(2) = 16 – 16 + 8 + 10 + 1 = 19

24.04) a) Resto = P(1) = 1 – 2 + 2 = 1

b) Resto = P(–1) = 2 – 4 + 6 = 4

c) Resto = $P(2) = 2^{14} - 64 \cdot 2^8$

$= 2^{14} - 2^6 \cdot 2^8$

$= 2^{14} - 2^{14}$

= 0

d) Resto = $P(1) = 1^n + 1$

= 1 + 1

= 2

24.05) A

Resto = $P(-1) = 5 \cdot (-1)^{2n} - 4 \cdot (-1)^{2n+1} - 2$

Lembre que 2n é um número par e 2n + 1 é um número ímpar para $n \in N$.

Assim,

P(–1) = 5 – 4 . (–1) – 2 = 5 + 4 – 2 = 7

24.06) A

Resto = $P(-1) = (-1)^{142} - 1 = 1 - 1 = 0$

24.07) D

Resto = $f(-a) = (-a)^n + a^n$

$= (-1)^n \cdot a^n + a^n$

$= a^n + a^n$ (**n** é par.)

$= 2a^n$

24.08) D

Resto = $P(-a) = (-a)^n + a^n$

$= (-1)^n \cdot a^n + a^n$

$$= \begin{cases} -a^n + a^n = 0, \text{ se } \mathbf{n} \text{ é ímpar.} \\ a^n + a^n = 2a^n, \text{ se } \mathbf{n} \text{ é par.} \end{cases}$$

24.09) E

P(2) = 0

~~16~~ – ~~2 . 8~~ + 2 – k = 0

k = 2

24.10) E

Resto = 0

P(–2) = 0

8 + 6 + m = 0 ⇒ m = –14

24.11) C

P(1) = 4

~~1~~ + a – ~~1~~ + a = 4

2a = 4

a = 2

24.12) A

P(1) = P(–2)

1 + q + 1 + 1 = –8 + 4q – 2 + 1

q + 3 = 4q – 9

12 = 3q

q = 4

24.13) A

Resto = 0

P(5) = 0

250 + 25m – 20 – 30 = 0

25m = –200

m = –8

24.14) P(2) = 48 – 8 + 2 + 1 = 43

24.15) D

P(–1) = 4 ⇒ –1 – p + q = 4

P(1) = 8 ⇒ 1 + p + q = 8

$$\begin{cases} -p + q = 5 \\ p + q = 7 \end{cases} \oplus$$

2q = 12

q = 6; p = 1

24.16) C

P(–1) = 0 ⇒ –a + b – 2 – 2 = 0

P(2) = 0 ⇒ 8a + 4b + 4 – 2 = 0

$$\begin{cases} -a + b = 4 \quad \cdot(-4) \\ 8a + 4b = -2 \end{cases}$$

$$\begin{cases} 4a - \cancel{4b} = -16 \\ 8a + \cancel{4b} = -2 \end{cases} \oplus$$

12a = –18

$$a = \frac{-18}{12}; a = \frac{-3}{2}$$

$$b = \frac{5}{2}$$

24.17) a) A(2) = 0 ⇒ 32 – 8 + 2a + b = 0

A(–1) = 9 ⇒ –4 – 2 – a + b = 9

$$\begin{cases} 2a + b = -24 \\ -a + b = 15 \quad \cdot(-1) \end{cases}$$

$$\begin{cases} 2a + \cancel{b} = -24 \\ a - \cancel{b} = -15 \end{cases} \oplus$$

3a = –39

a = –13; b = 2

b) $A(x) = 4x^3 - 2x^2 - 13x + 2$

A(–3) = –108 – 18 + 39 + 2 = –85

24.18) f(–1) = f(1)

$-2 + \cancel{3} - k - \cancel{2} = 2 + \cancel{3} + k - \cancel{2}$

$-4 = 2k$

$k = -2$

$\Rightarrow f(x) = 2x^3 + 3x^2 - 2x - 2$

Divisão por $(x + 1) \cdot (x - 1) = x^2 - 1$

$$\begin{array}{r|l} 2x^3 + 3x^2 - 2x - 2 & x^2 - 1 \\ \cancel{-2x^3} + 2x & \hline & 2x + 3 \\ \hline 3x^2 - 2 & \\ \cancel{-3x^2} + 3 & \\ \hline 1 & \end{array}$$

Resto = 1

24.19) B

$$\begin{array}{r|l} f(x) & x^2 - 3x + 1 \\ & \hline & x + 1 \\ 2x + 1 & \end{array}$$

$\Rightarrow f(x) = (x^2 - 3x + 1) \cdot (x + 1) + 2x + 1$

O resto da divisão de **f** por x + 1 é:

$f(-1) = (1 + 3 + 1) \cdot (-1 + 1) - 2 + 1 = -1$

24.20) D

$$\begin{array}{r|l} f(x) & (x - 1) - (x + 2) \\ & \hline & Q(x) \\ 2x - 1 & \end{array}$$

$f(x) = (x - 1) \cdot (x + 2) \cdot Q(x) + 2x - 1$

O resto da divisão de f(x) por x + 2 é

$f(-2) = (-2 - 1) \cdot (-2 + 2) \cdot Q(-2) - 4 - 1 = -5$

24.21) P(–1) = 0

$$\begin{array}{r|l} P(x) & x^2 + 1 \\ & \hline & x^2 - 4 \end{array}$$

R(x)=ax + b

$P(x) = (x^2 + 1) \cdot (x^2 - 4) + ax + b$

$P(-1) = 0 \Rightarrow (1 + 1) \cdot (1 - 4) - a + b = 0$

$-6 - a + b = 0$

$-a + b = 6$ (I)

Além disso,

$R(2) = 9$

$\Rightarrow 2a + b = 9$ (II)

De I e II, temos:

$$\begin{cases} -a + b = 6 & .(-1) \\ 2a + b = 9 \end{cases}$$

$$\begin{cases} a - \cancel{b} = -6 \\ 2a + \cancel{b} = 9 \end{cases} \oplus$$

$3a = 3$

$a = 1; b = 7 \Rightarrow R(x) = x + 7$

Logo, $P(x) = (x^2 + 1) \cdot (x^2 - 4) + x + 7 =$

$= x^4 - 4x^2 + x^2 - 4 + x + 7$

$= x^4 - 3x^2 + x + 3$

24.22) P(x) | $x^2 - 2x - 8$ / Q(x)

P(x)=ax + b

$P(x) = (x^2 - 2x - 8) \cdot Q(x) + ax + b$

$P(4) = 0 \Rightarrow \cancel{(16 - 8 - 8) \cdot Q(4)} + 4a + b = 0$

$4a + b = 0$ (I)

$P(-2) = 12 \Rightarrow \cancel{(4 + 4 - 8) \cdot Q(-2)} - 2a + b = 12$

$-2a + b = 12$ (II)

De I e II, temos:

$$\begin{cases} 4a + b = 0 \\ -2a + b = 12 & .(-1) \end{cases}$$

$$\begin{cases} 4a + \cancel{b} = 0 \\ 2a - \cancel{b} = -12 \end{cases} \oplus$$

$6a = -12$

$a = -2; b = 8$

Logo, $R(x) = -2x + 8$.

24.23) B P(x) | $(x - 2)(x + 2)$ / Q(x)

R(x)=ax + b

$P(x) = (x - 2) \cdot (x + 2) \cdot Q(x) + ax + b$

$P(2) = 6 \Rightarrow \cancel{(2 - 2) \cdot (2 + 2) \cdot Q(2)} + 2a + b = 6$

$2a + b = 6$ (I)

$P(-2) = 10 \Rightarrow \cancel{(-2 - 2) \cdot (-2 + 2) \cdot Q(-2)} - 2a + b = 10$

$-2a + b = 10$ (II)

De I e II vem, temos:

$$\begin{cases} \cancel{2a} + b = 6 \\ -\cancel{2a} + b = 10 \end{cases} \oplus$$

$2b = 16$

$b = 8; a = -1$

Logo, $R(x) = -x + 8$.

24.24) $P(x) \;\big|\; (x-1)^2$ — $Q(x)$

$\underbrace{R(x)}$

$P(x) = (x-1)^2 \cdot Q(x) + R(x)$
$R(x)$ dividido por $x - 1$ dá resto 3.
$R(1) = 3$

Assim, $P(1) = \cancel{(1-1)^2 \cdot Q(1)} + R(1) = 3$

24.25) $P(x) = x^{100} + x^{99} + 1 \;\big|\; x^2 - 1$ — $Q(x)$

$\underbrace{R(x) = ax + b}$

$\Rightarrow P(x) = x^{100} + x^{99} + 1 = (x^2 - 1) \cdot Q(x) + ax + b$
Observe que
$P(1) = 1 + 1 + 1 = 3$ e
$P(-1) = 1 - 1 + 1 = 1$
Por outro lado:

$P(1) = \cancel{(1^2 - 1) \cdot Q(1)} + a + b = 3$

$a + b = 3$ (I)

$P(-1) = \cancel{(1-1) \cdot Q(-1)} - a + b = 1$

$-a + b = 1$ (II)
De I e II, temos:

$$\begin{cases} a + b = 3 \\ -a + b = 1 \end{cases} \oplus$$

$2b = 4 \Rightarrow b = 2; a = 1$
Assim, $R(x) = x + 2$.

24.26) $P(x) = ax^3 + bx^2 + cx + d$
$P(1) = 0 \Rightarrow a + b + c + d = 0$ (I)
$P(-1) = 6 \Rightarrow -a + b - c + d = 6$ (II)
$P(2) = 6 \Rightarrow 8a + 4b + 2c + d = 6$ (III)
$P(-2) = 6 \Rightarrow -8a + 4b - 2c + d = 6$ (IV)
Somando I + II: $2b + 2d = 6$. (–1)
Somando III + IV: $8b + 2d = 12$

$$\begin{cases} -2b - \cancel{2d} = -6 \\ 8b + \cancel{2d} = 12 \end{cases} \oplus$$

$6b = 6$
$b = 1; d = 2$
Substituindo em I e II, temos:

$$\begin{cases} a + c = -3 \\ 8a + 2c = 0 \end{cases} \cdot (-2)$$

$$\begin{cases} -2a - \cancel{2c} = 6 \\ 8a + \cancel{2c} = 0 \end{cases} \oplus$$

$6a = 6$
$a = 1; c = -4$
Logo, $P(x) = x^3 + x^2 - 4x + 2$

24.27) E
$P(1) = R_1$; $P(-1) = R_2$

$P(x) \;\big|\; x^2 - 1$ — $Q(x)$

$\underbrace{R(x) = ax + b}$

$P(x) = (x^2 - 1) \cdot Q(x) + ax + b$

$P(1) = R_1 \Rightarrow \cancel{(1-1) \cdot Q(1)} + a + b = R_1$

$P(-1) = R_2 \Rightarrow \cancel{(1-1) \cdot Q(-1)} - a + b = R_2$

$$\begin{cases} a + b = R_1 \\ -a + b = R_2 \end{cases} \oplus$$

$2b = R_1 + R_2$

$b = \dfrac{R_1 + R_2}{2}$

$R(x) = ax + b$

$R(0) = b = \dfrac{R_1 + R_2}{2}$