Matemática E – Extensivo – V. 2

Resolva

Aula 5

5.01) $a_1 = 32$

$q = \frac{1}{2}$

$a_{10} = a_1 \cdot q^9$

$a_{10} = 32 \cdot \left(\frac{1}{2}\right)^9 = 2^5 \cdot \frac{1}{2^9} = 2^{-4}$

$P_{10} = \sqrt{(32 \cdot 2^{-4})^{10}}$

$P_{10} = (2^5 \cdot 2^{-4})^5 = (2^2)^5 = 32$

5.02) P.A. $(x - r, x, x + r)$

$x - \cancel{r} + x + x + \cancel{r} = 12$

$3x = 12$

$x = 4$

P.A. $(4 - r, 4, 4 + r)$

P.G. $(4 - r, 4, 6 + r)$

$4^2 = (4 - r) \cdot (6 + r)$

$16 = 24 + 4r - 6r - r^2$

$r^2 + 2r - 8 = 0$

$r' = -4$

$r'' = 2$

Como a P.A. cresce, r = 2.

P.G. (2, 4, 8)

Produto: 2 . 4 . 8 = 64

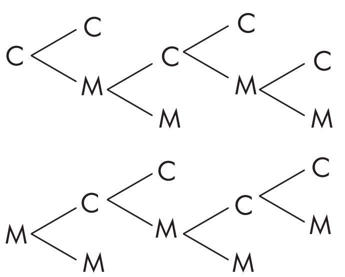
5.03) D

$\left(\frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \frac{1}{14}, \ldots\right)$

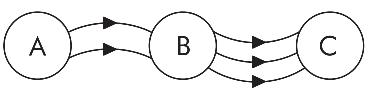
É uma progressão harmônica, pois os inversos (5, 8, 11, 14, ...) formam uma P.A.

Aula 6

6.01) J_1 J_2 J_3 J_4 J_5



6.02)



2 . 3 = 6

6.03) $\underset{1}{\overline{C}} \cdot \underset{3}{\overline{A}} \cdot \underset{4}{\overline{B}} = 12$

6.04) letras algarismos

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 10 = 1560000$

Aula 7

7.01) C

$\overline{5} \cdot \overline{4} \cdot \overline{3} \cdot \overline{2} \cdot \overline{1} = 5!$

7.02) a) $\frac{7!}{5!} = \frac{7 \cdot 6 \cdot \cancel{5!}}{5!} = 42$

b) $6! - 5! = 720 - 120 = 600$

c) $\frac{101! - 100!}{100!} = \frac{101 \cdot 100! - 100!}{100!} =$

$= \frac{\cancel{100!}(101 - 1)}{\cancel{100!}} = 100$

7.03) $\frac{(n + 1)!}{(n - 1)!} = 72$

$\frac{(n + 1) \cdot n \cdot \cancel{(n - 1)!}}{\cancel{(n - 1)!}} = 72$

$n^2 + n - 72 = 0$

$n' = 8$

~~$n'' = -9$~~ S = {8}

7.04) D

$20 \cdot 18 \cdot 16 \cdot 14 \cdot \ldots \cdot 6 \cdot 4 \cdot 2$

$= 10 \cdot 2 \cdot 9 \cdot 2 \cdot 8 \cdot 2 \cdot 7 \cdot 2 \cdot \ldots \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 1$

$= 10 \cdot 9 \cdot 8 \cdot 7 \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \cdot 2^{10}$

$= 2^{10} \cdot 10!$

Aula 8

8.01) $\overline{8} \cdot \overline{7} \cdot \overline{6} = 336$

ou $A_{8,3} = 336$

8.02) $C_{12,3} = \frac{12!}{3!9!} = \frac{12 \cdot 11 \cdot 10 \cdot \cancel{9!}}{3 \cdot 2 \cdot 1 \cdot \cancel{9!}} = 220$

8.03) a) $\underline{5} \cdot \underline{5} \cdot \underline{5} = 125$

b) $\underline{5} \cdot \underline{4} \cdot \underline{3} = 60$ ou $A_{5,3} = 60$

c) $\underset{\uparrow}{\underline{4}} \cdot \underset{\uparrow}{\underline{3}} \cdot \underset{\uparrow}{\underline{2}} = 24$

d) $\underline{2}\ \underline{4} \cdot \underline{3} = 12$

$\underline{3}\ \underline{4} \cdot \underline{3} = 12$

$\underline{5}\ \underline{4} \cdot \underline{3} = 12$

$\underline{5}\ \underline{4} \cdot \underline{3} = 12$

$\underline{7}\ \underline{2}\ \underline{3} = 3$

$\underline{7}\ \underline{3}\ \underline{3} = 3$

Total: 66

8.04) a) $C_{10,2} = \dfrac{10!}{2!8!} = \dfrac{10\,.\,9\,.\,\cancel{8!}}{2!\,\cancel{8!}} = 45$

b) $C_{10,3} = \dfrac{10!}{3!7!} = \dfrac{\cancel{10}^{5}\,.\,\cancel{9}^{3}\,.\,8\,.\,\cancel{7!}}{\cancel{3}\,.\,\cancel{2}\,.\,1\,.\,\cancel{7!}} = 120$

c) $C_{10,2} - 10$
$= 45 - 10$
$= 35$

Testes

Aula 5

5.01) $a_{10} = \dfrac{1}{16} \,.\, 2^9 = 2^{-4} \,.\, 2^9 = 2^5$

$P_{10} = \sqrt{(2^{-4}.2^5)^{10}}$

$P_{10} = 32$

5.02) 15

01. **Verdadeira.**

$a_{20} = a_1 + 19r$
$30 = -8 + 19r$
$38 = 19r$
$r = 2$

02. **Verdadeira.**

P.A. (5, 8, ..., 41)
$41 = 5 + (n - 1)\,.\,3$
$41 = 5 + 3n - 3$
$39 = 3n$
$n = 13$

$S_n = \dfrac{(5 + 41)\,.\,13}{2} = \dfrac{\cancel{46}^{23}\,.\,13}{\cancel{2}} = 299$

04. **Verdadeira.**

$a_7 = a_3 \,.\, q^4$

$\dfrac{\cancel{3}}{16} = \cancel{3}\,.\,q^4$

$\dfrac{1}{16} = q^4$

$q = \pm\dfrac{1}{2}$

$a_3 = a_1 \,.\, q^2$

$3 = a_1 \,.\, \dfrac{1}{4}$

$a_1 = 12$

08. **Verdadeira.**

$S_\infty = \dfrac{5}{1-\frac{1}{2}} = \dfrac{5}{\frac{1}{2}} = 10$

5.03) P.G. (x, $5x$, 5^2x, 5^3x, 5^4x, 5^5x)

$x \,.\, 5^5 \,.\, x = 12500$

$x^2 = \dfrac{12500}{3125}$

$x^2 = 4$

$x = 2$

$a_3 = 5^2 \,.\, x = 50$

5.04) 13

01. **Correta.**

P.A. (21, 28, ..., 1197)
$1197 = 21 + (n - 1)\,.\,7$
$1197 = 21 + 7n - 7$
$1183 = 7n$
$n = 169$

02. **Incorreta.**

$a_n = 4n + 7$
$a_{10} = 4\,.\,10 + 7 = 47$

04. **Correta.**

Uma seqüência só é simultaneamente P.A. e P.G. se for uma seqüência constante. Veja:

P.A. (a, b, c) $\Rightarrow b = \dfrac{a + c}{2}$

P.G. (a, b, c) $\Rightarrow b^2 = a\,.\,c$

$\left(\dfrac{a + c}{2}\right)^2 = a\,.\,c$

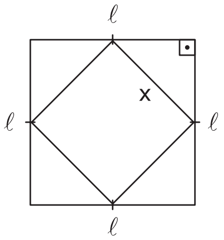
$\dfrac{a^2 + 2ac + c^2}{4} = ac$

$a^2 + c^2 + 2ac = 4ac$
$a^2 - 2ac + c^2 = 0$
$(a - c)^2 = 0$
$a - c = 0$
$a = c$

Como $b = \dfrac{a + c}{2}$, temos: $b = \dfrac{a + a}{2}$

$b = a$
Assim, $a = b = c$.

08. **Correta**.



$$x^2 = \left(\frac{\ell}{2}\right)^2 + \left(\frac{\ell}{2}\right)^2$$

$$x^2 = \frac{\ell^2}{4} + \frac{\ell^2}{4}$$

$$x^2 = \frac{2\ell^2}{4}$$

$$x = \frac{\ell\sqrt{2}}{2}$$

Área do 1º: ℓ^2

Área do 2º: $x^2 = \dfrac{\ell^2}{2}$

P.G. $\left(\ell^2, \dfrac{\ell^2}{2}, ...\right)$

$$q = \frac{1}{2}$$

5.05) A

$$\begin{cases} a_4 + a_6 = 160 \\ a_7 + a_9 = 1280 \end{cases}$$

$$\begin{cases} a_1 \,.\, q^3 + a_1 \,.\, q^5 = 160 \\ a_1 \,.\, q^6 + a_1 \,.\, q^8 = 1280 \end{cases}$$

$$\left\{ \frac{\cancel{a_1} \,.\, \cancel{q^3}\ \cancel{(1+q^2)}}{\cancel{a_1} \,.\, q^{\cancel{6}}\ \cancel{(1+q^2)}} = \frac{160}{1280} \quad (\div) \right.$$

$$\frac{1}{q^3} = \frac{1}{8}$$

$q = 2$

$a_1 \,.\, q^3 + a_1 \,.\, q^5 = 160$
$8a_1 + 32a_1 = 160$
$40a_1 = 160$
$a_1 = 4$

(4, 8, 16, 32, 64, ...)
Assim, pode ser uma P.G. de razão $q = 2$.

5.06) P.A. (1, 2, 3, 4, 5, ...) $r = 1$
$a_{30} = 1 + 29 \,.\, 1 = 30$

$$S_{30} = \frac{(1 + 30) \,.\, 30}{2} = 465$$

$465 - 300 = 165$

5.07) a) 1º) P.G. (10000 m, 9000 m, 8100 m, 7290 m, 6561 m)
2º) P.A. (8000 m, 8200 m, 8400 m, 8600 m, 8800 m)
Até o 4º dia eles percorreram:
1º) 10000 + 9000 + 8100 + 7290 = 34390 m
2º) 8000 + 8200 + 8400 + 8600 = 33200 m
Isso significa que o segundo ainda não alcançou o primeiro após 5 dias:
1º) 34390 + 6561 = 40951 m
2º) 33200 + 8800 = 42000 m
Portanto, o 2º alcança o 1º após 5 dias.

b) 1º) P.G. (12000 m, 10800 m, 9720 m; 8748 m, 7873,2 m, 7085,9 m; 6377,3 m, 5739,6 m, 5165,6 m)
2º) P.A. (8000 m, 8100 m, 8200 m, 8300 m, 8400 m, 8500 m, 8700 m, 8800 m)
Até o 8º dia, percorreram:
1º) S_8 = 12000 + 10800 + 9720 + ... + 5739,6 = 68344 m
2º) S_8 = 8000 + 8100 + 8200 + ... 8700 = 66800 m
Percebemos assim que o 2º alcança o 1º após 9 dias.
Observação: Nesse item as dificuldades apresentadas estão nos cálculos da primeira seqüência.

5.08) 15

01. **Verdadeira**.
P.A. (56, 63, ..., 497)
$497 = 56 + (n - 1) \,.\, 7$
$497 = 56 + 7n - 7$
$448 = 7n$
$n = 64$

02. **Verdadeira**.

$$S_n = \frac{(a_1 + a_n) \,.\, n}{2}\ ;\ n = 10$$

$$155 = \frac{(x + 1 + x + 28) \,.\, \overset{5}{\cancel{10}}}{\cancel{2}}$$

$31 = 2x + 29$
$x = 1$

04. **Verdadeira**.

$$q = \frac{2}{\sqrt{2}} \,.\, \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$a_8 = \sqrt{2} \,.\, (\sqrt{2}\,)^7 = (\cancel{\sqrt{}}2)^{\cancel{8}} = 2^4 = 16$$

08. **Verdadeira.**

$q = \frac{2}{3}$

$S_\infty = \frac{\frac{1}{3}}{1 - \frac{2}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$

5.09) E

P.G. (1, 2, 4, 8)

Soma = 15

P.A. (1, 1 + r, 1 + 2r, 1 + 3r)

Soma: 1 + 1 + r + 1 + 2r + 1 + 3r = 15

6r = 11

$r = \frac{11}{6}$

5.10) P.A. $(a_1, ..., a_5, ..., a_{21})$

P.A. $(a_1, ..., a_1 + 4r, ..., a_1 + 20r)$

P.G. $(a_1, a_1 + 4r, a_1 + 20r)$

$(a_1 + 4r)^2 = a_1 \cdot (a_1 + 20r)$

$\cancel{a_1^2} + 8a_1r + 16r^2 = \cancel{a_1^2} + 20a_1r$

$16r^2 - 12a_1r = 0$

$4r \cdot (4r - 3a_1) = 0$

$4r = 0$

~~r = 0~~ (Os termos são diferentes.)

ou $4r - 3a_1 = 0$

$r = \frac{3a_1}{4}$

Assim, a P.G. é:

P.G. $(a_1, a_1 + 3a_1, a_1 + 15a_1)$

P.G. $(a_1, 4a_1, 16a_1)$

q = 4

5.11) 07

01. **Correta.**

P.A. (5 – r, 5, 5 + r)

P.G. (5 + r, 4, 5 – r)

$4^2 = (5 + r) \cdot (5 - r)$

$16 = 25 - r^2$

r = 3

(P.A. é crescente.)

P.A. (2, 5, 8, ...)

$a_{10} = 2 + 9 \cdot 3 = 29$

$S_{10} = \frac{(2 + 29) \cdot 10}{2} = 155$

02. **Correta.**

	Nov.	Dez.	Jan.	Fev.
Receita (P.G.)	300	360	432	518,4
Despesa (P.A.)	350	405	460	515

04. **Correta.**

Aumento: $\left(4, 2, 1, \frac{1}{2}, ...\right)$

$S_\infty = \frac{4}{1 - \frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8$ kg

Nunca atingirá 68 kg, pois não viverá infinitos anos.

08. **Incorreta.**

Raios: P.G. $(r, rq, rq^2, rq^3, ...)$

Áreas: $(\pi r^2, \pi r^2q^2, \pi r^2q^4, \pi r^2q^6, ...)$

Razão: $\frac{\pi r^2 q^2}{\pi r^2} = q^2$

5.12) A

P.A. (9, x, y)

$x = \frac{9 + y}{2}$

P.G. $\left(x, y, \frac{3}{2}\right)$

$y^2 = x \cdot \frac{3}{2}$

$y^2 = \frac{9 + y}{2} \cdot \frac{3}{2}$

$y^2 = \frac{27 + 3y}{4}$

$4y^2 - 3y - 27 = 0$

y' = 3

~~$y'' = -\frac{9}{4}$~~

$y = 3$ e $x = \frac{9 + 3}{2} = 6$

5.13) P.A. (x, 2y, 3z)

$2y = \frac{x + 3z}{2}$

$4y = x + 3z$

$z = \frac{4y - x}{3}$

P.G. (x, y, z)

$y^2 = x \cdot z$

$y^2 = x \cdot \frac{4y - x}{3}$

$3y^2 = 4yx - x^2$

$3y^2 - 4yx + x^2 = 0$

Vamos resolver essa equação do 2º grau considerando **y** como variável. Assim, a = 3, b = –4x e c = x^2.

$\Delta = (-4x)^2 - 4 \cdot 3 \cdot x^2 = 16x^2 - 12x^2 = 4x^2$

$y = \frac{4x \pm \sqrt{4x^2}}{6}$

$$y = \frac{4x \pm 2x}{6} \begin{cases} y' = \frac{6x}{6} = x \\ y'' = \frac{2x}{6} = \frac{x}{3} \end{cases}$$

Como y = x, então:

$$y = \frac{x}{3}$$

$$\frac{y}{x} = \frac{1}{3}$$

P.G. (x, y, z)

Razão: $q = \frac{y}{x} = \frac{1}{3}$

5.14) D
P.A. (10742, 9310, 7878, ...)
r = 9310 – 10742 = –1432
r = 7878 – 9310 = –1432

5.15) C
$S_1 = 3 + 7 + 11 + ... + 43$ (P.A.)
$a_n = a_1 + (n - 1) . r$
$43 = 3 + (n - 1) . 4$
$40 = 4n - 4$
$n = 11$

$$S_1 = \frac{(3 + 43) . 11}{2} = 253$$

$S_2 = 1 + 2 + 4 + ... + 128$ P.G.

$$S_2 = \frac{a_n . q - a_1}{q - 1} = \frac{128 . 2 - 1}{2 - 1} = 255$$

$\Rightarrow S_2 - S_1 = 2$

5.16) B
P.A. (x, y, z)

$$y = \frac{x + z}{2}$$

2y = x + z

$$\text{P.G.} \left(\frac{1}{x}, \frac{1}{y}, \frac{1}{x + z}\right)$$

$$\frac{1}{y^2} = \frac{1}{x} . \frac{1}{x + z}$$

$$\frac{1}{\cancel{y^2}} = \frac{1}{x} . \frac{1}{2\cancel{y}}$$

$$\frac{1}{y} = \frac{1}{2x}$$

y = 2x
2y = x + z
2 . 2x = x + z
z = 3x
y + z = 2x + 3x = 5x

5.17) D

$a_n = (1, 3, 7, 13, 21, ...)$ (+2, +4, +6, +8, ...)
Observe que a seqüência $b_n = (2, 4, 6, 8, ...)$ é uma P.A. Além disso, veja:
2 + 4 + 1 = 7
$b_1 + b_2 + 1 = a_3$
2 + 4 + 6 + 1 = 13
$b_1 + b_2 + b_3 + 1 = a_4$
Dessa forma, $a_{11} = b_1 + b_2 + ... + b_{10} + 1$.
$b_{10} = 2 + (10 - 1) . 2 = 20$

$$S_{10} = \frac{(2 + 20) . 10}{2} = 110$$

$a_{11} = 110 + 1 = 111$

5.18) C
P.A. (20 mm, 21 mm, 22 mm, ..., 150 mm)
Diâmetros: (40, 42, 44, ..., 300)
P.A. r = 2

5.19) a) 13º dia : 13
14º dia : 13 (+)
15º dia : 13
⋮
40º dia : 13
28 . 13 = 364

b) Cada morador colocou a seguinte quantidade de enfeites:

Enfeites		nº de dias		Total
1	.	40	=	40
2	.	39	=	78
3	.	38	=	114
4	.	37	=	148
⋮		⋮		⋮
19	.	22	=	418
20	.	21	=	420
21	.	20	=	420
22	.	19	=	418
⋮		⋮		⋮
39	.	2	=	78
40	.	1	=	40

O número máximo de enfeites é 420, que serão colocados pelos moradores de número 20 e 21.

5.20) C

P.A. (2, 5, 8, 11, 14, 17, 20, ..., 332) $r = 3$

P.A. (7, 12, 17, 22, 27, ..., 157) $r = 5$

Os termos em comum são:

(17, 32, 47, 62, ..., 152) $r = 15$

Eles formam uma P.A. cuja razão é o m.m.c. entre as razões dadas.

$a_n = a_1 + (n - 1) . r$

$152 = 17 + (n - 1) . 15$

$152 = 17 + 15n - 15$

$150 = 15n$

$n = 10$

5.21) P.A. $(x - r, x, x + r)$

$x - r + x + x + r = 36$

$3x = 36$

$x = 12$

P.A. $(12 - r, 12, 12 + r)$

P.G. $(12 - r, 12, 18 + r)$

$12^2 = (12 - r) . (18 + r)$

$144 = 216 + 12r - 18r - r^2$

$r^2 + 6r - 72 = 0$

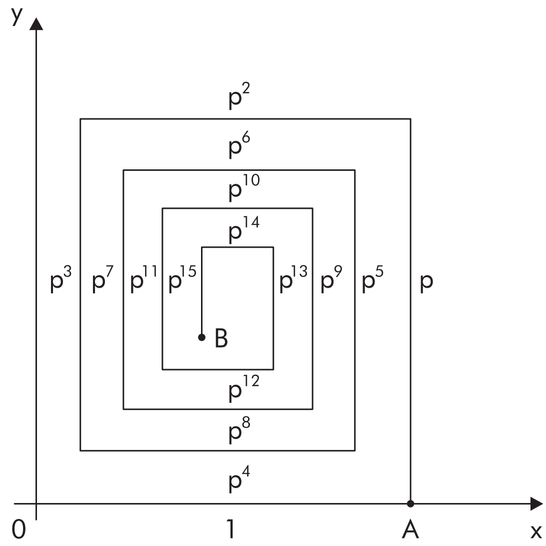
$r' = 6$

~~$r'' = -12$~~

(P.A. crescente)

Os números são: P.A. (6, 12, 18)

5.22) D



A abcissa do ponto B é dada pela soma:

$S = 1 - p^2 + p^4 - p^6 + p^8 - p^{10} + p^{12} - p^{14}$

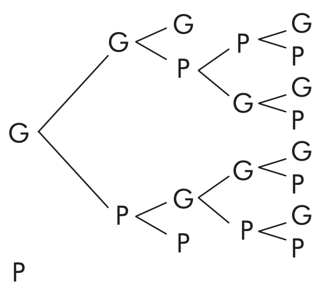
Note que $(1, -p^2, p^4, ..., -p^{14})$ é uma P.G. de razão $-p^2$. Logo,

$$S = \frac{a_1 . (q^n - 1)}{q - 1}$$

$$S = \frac{1 . ((-p^2)^8 - 1)}{-p^2 - 1} = \frac{p^{16} - 1}{-p^2 - 1} = \frac{1 - p^{16}}{1 + p^2}$$

Aula 6

6.01) 1ª 2ª 3ª 4ª 5ª



Número de maneiras: 11

6.02) a) $\underset{\underset{1}{\uparrow}}{\underline{\text{motorista}}} \cdot \underset{\underset{4}{\uparrow}}{\underline{\text{cobrador}}} = 4$

b) Para cada motorista, há 4 cobradores. Logo, podem ser formadas 4 duplas.

c) $\underline{\ \ }\ \underline{\ \ }$
$3 \cdot 4 = 12$

6.03) 6 . 8 = 48

6.04) 8 . 5 = 40

6.05) 5 . 6 = 30

6.06) 80 . 90 = 7200

6.07) 8 . 7 = 56

6.08) 10 . 12 . 5 = 600

6.09) a) $\frac{\text{letra}}{3} \cdot \frac{\text{nº}}{4} = 12$

b $\frac{\text{nº}}{4} \cdot \frac{\text{nº}}{3} = 12$

6.10) $\frac{x \to A}{8} \cdot \frac{A \to y}{7} = 56$

$\frac{x \to B}{12} \cdot \frac{B \to y}{8} = 96$

Total: 56 + 96 = 152

6.11) D

$\frac{A \to B}{3} \cdot \frac{B \to C}{4} \cdot \frac{C \to B}{3} \cdot \frac{B \to A}{2} = 72$

6.12) a) $\underline{5} \cdot \underline{5} \cdot \underline{5} = 125$

b) $\underline{5} \cdot \underline{4} \cdot \underline{3} = 60$

6.13) $\underline{2}\ \underline{3}\ \underline{7}\ \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 10000$

6.14)

urna A		urna B		
↑		↑		
100	.	50	=	5000

urna B		urna A		
↑		↑		
50	.	100	=	5000

Total: 10000

6.15) pessoa A ↑ 5 · pessoa B ↑ 4 = 20

6.16) D

Verde, amarelo, x, y, z

Sul	Sudeste	Centro-Oeste	Nordeste	Norte	
2 .	4 .	3 .	2 .	1	= 48

6.17) $\underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} = 2^{10} = 1024$

6.18) $\underline{3} \cdot \underline{3} \cdot \underline{3} \cdot \underline{3} \cdot \underline{3} \cdot \underline{3} = 3^6 = 729$

6.19)

Calça	Paletó
1	24
2	12
3	8
4	6
6	4
8	3
12	2
24	1

Número mínimo: 4 + 6 = 10

6.20) 11

01. **Correto**. Por exemplo, com as cores azul, branco e cinza:

Sul	Sde	C – O	Nde	Nte
A	B	C	A	B

02. **Correto**.

Sul	Sde	C – O	Nde	Nte
5 .	4 .	3 .	2 .	1

04. **Incorreto**.

Sul	Sde	C – O	Nde	Nte
1 .	4 .	3 .	5 .	3

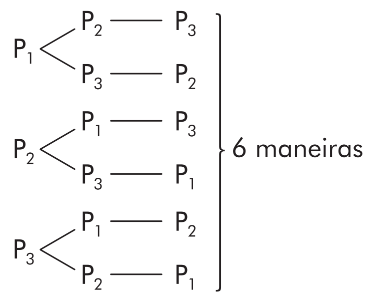
08. **Correto**.

Sul	Sde	C – O	Nde	Nte
1 .	1 .	3 .	5 .	4

6.21) a) $\underset{4}{\underline{P}} \cdot \underset{3}{\underline{S}} \cdot \underset{2}{\underline{Q}} \cdot \underset{2}{\underline{R}} = 48$

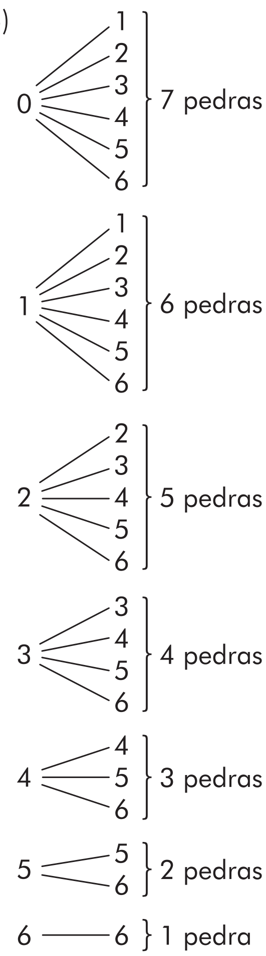
b) $\underset{4}{\underline{P}} \cdot \underset{1}{\underline{S}} \cdot \underline{3} \cdot \underline{3} = 36$

6.22) C_1 C_2 C_3



6.23) $\underset{3}{\underline{M_1}} \cdot \underset{3}{\underline{M_2}} \cdot \underset{3}{\underline{M_3}} \cdot \underset{3}{\underline{M_4}} \cdot \underset{3}{\underline{M_5}} = 3^5 = 243$

6.24)

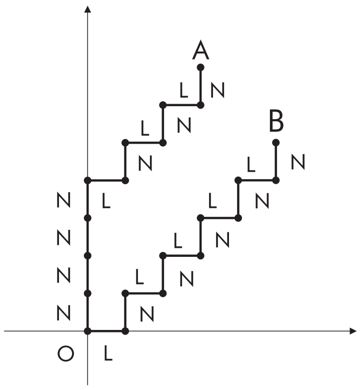


Total: 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28

6.25) $\underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} = 32$

6.26) D

Exemplos:



Trajetória OA: NNNNLNLNLN

Trajetória OB: LNLNLNLNLN

Total de trajetória:

$\underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} = 2^{10}$

6.27) $\underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 175760000$

Aula 7

7.01) a) $\dfrac{10!}{8!} = \dfrac{10 \,.\, 9 \,.\, \cancel{8!}}{\cancel{8!}} = 90$

b) $\dfrac{26!}{23!} = \dfrac{26 \,.\, 25 \,.\, 24 \,.\, \cancel{23!}}{\cancel{23!}} = 15600$

c) $\dfrac{n!}{(n-1)!} = \dfrac{n \,.\, \cancel{(n-1)!}}{\cancel{(n-1)!}} = n$

d) $\dfrac{(n-2)!}{(n-3)!} = \dfrac{(n-2) \,.\, \cancel{(n-3)!}}{\cancel{(n-3)!}} = n-2$

e) $\dfrac{(n+1)!}{(n-1)!} = \dfrac{(n+1)n \,.\, \cancel{(n-1)!}}{\cancel{(n-1)!}} = n^2 + n$

7.02) $\dfrac{(n+1)! + n!}{(n+2)!} = \dfrac{(n+1) \,.\, \cancel{n!} + \cancel{n!}}{(n+2) \,.\, (n+1) \,.\, \cancel{n!}}$

$= \dfrac{n+1+1}{(n+2) \,.\, (n+1)} = \dfrac{\cancel{n+2}}{\cancel{(n+2)} \,.\, (n+1)} = \dfrac{1}{n+1}$

7.03) A

$(n-6)! = 720$

$(n-6)! = 6!$

$n - 6 = 6$

$n = 12$

7.04) C

$2 < n! < 24$

$2! < n! < 4!$

$2 < n < 4$

$n = 3$

7.05) $(x!)^2 + 5 \,.\, x! - 6 = 0$

Com $x! = y$, temos:

$y^2 + 5y - 6 = 0$

$y' = -6$

$y'' = 1$

$x! = 1$

$x = 0$ ou $x = 1$

$x! = -6$ (Não existe!)

$S = \{0,1\}$

7.06) $(x+1)! = x! + 6x \div (x!)$

$\dfrac{(x+1)!}{x!} = \dfrac{\cancel{x!}}{\cancel{x!}} + \dfrac{6x}{x!}$

$\dfrac{(x+1) \,.\, \cancel{x!}}{\cancel{x!}} = 1 + \dfrac{6x}{x!}$

$x + \cancel{1} = \cancel{1} + \dfrac{6x}{x!}$

$x \,.\, x! = 6x$

$x \,.\, x! - 6x = 0$

$x \,.\, (x! - 6) = 0$

$x = 0$

ou

$x! - 6 = 0$

$x! = 6$

$x = 3$

Soma: $0 + 3 = 3$

Produto: $0 \,.\, 3 = 0$

7.07) A

$\dfrac{(n+2)!\,(n-2)!}{(n+1)!\,(n-1)!} = 4$

$\dfrac{(n+2)\ \cancel{(n+1)!}\ \cancel{(n-2)!}}{\cancel{(n+1)!}\,(n-1)\ \cancel{(n-2)!}} = 4$

$\dfrac{n+2}{n-1} = 4$

$n + 2 = 4n - 4$

$6 = 3n$

$2 = n$

7.08) $\dfrac{n!}{(x-2)!} + 25 = \dfrac{(2n)!}{2(2n-2)!}$

$\dfrac{n \,.\, (n-1) \,.\, \cancel{(n-2)!}}{\cancel{(n-2)!}} + 25 = \dfrac{(2n) \,.\, (2n-1) \,.\, \cancel{(2n-2)!}}{\cancel{2}\ \cancel{(2n-2)!}}$

$n^2 - \cancel{n} + 25 = 2n^2 - \cancel{n}$

$0 = n^2 - 25$

$n' = 5;\ n'' = -5$

$S = \{5\}$

7.09) D

$(5x - 7)! = 1$

$5x - 7 = 0$

$x = \dfrac{7}{5}$

ou

$5x - 7 = 1$

$x = \dfrac{8}{5}$

Soma: $\dfrac{7}{5} + \dfrac{8}{5} = \dfrac{15}{5} = 3$

7.10) $\dfrac{(n+1)!\,n!}{(n-1)!} = 6n$

$\dfrac{(n+1)!\,\cancel{n}\,\cancel{(n-1)!}}{\cancel{(n-1)!}} = 6\cancel{n}$

$(n + 1)! = 6$

$n + 1 = 3$

$n = 2$

7.11) E

$\dfrac{n!}{n(n+1)!} = \dfrac{\cancel{n!}}{n(n+1)\,\cancel{n!}} = \dfrac{1}{n(n+1)}$

7.12) B

$\dfrac{n! + (n-1)!}{(n+1)!}$

$= \dfrac{n\,.\,\cancel{(n-1)!} + \cancel{(n-1)!}}{(n+1)\,.\,n\,.\,\cancel{(n-1)!}} = \dfrac{\cancel{n+1}}{\cancel{(n+1)}\,.\,n} = \dfrac{1}{n}$

7.13) E

$[(m + 2)! - (m + 1)!]\ m!$

$= [(m + 2)\,.\,(m + 1)! - (m + 1)!]\,.\,m!$

$= [(m + 1)!\ (m + 2 - 1)]\ m!$

$= (m + 1)!\ \underbrace{(m + 1)\,.\,m!}$

$= (m + 1)!\ (m + 1)!$

$= [(m + 1)!]^2$

7.14) E

$\dfrac{(n+1)! - n!}{(n-1)! + n!}$

$= \dfrac{(n+1)\,.\,n\,.\,(n-1)! - n(n-1)!}{(n-1)! + n\,.\,(n-1)!}$

$= \dfrac{\cancel{(n-1)!}\ [(n+1)\,.\,n - n]}{\cancel{(n-1)!}\ [1+n]} = \dfrac{n^2 + \cancel{n} - \cancel{n}}{n+1} = \dfrac{n^2}{n+1}$

7.15) $\dfrac{1}{(n-1)!} - \dfrac{1}{n!} = \dfrac{n! - (n-1)!}{(n-1)!\,n!}$

$= \dfrac{n\,.\,(n-1)! - (n-1)!}{(n-1)!\,n!} = \dfrac{\cancel{(n-1)!}\ [n-1]}{\cancel{(n-1)!}\ n!} = \dfrac{n-1}{n!}$

7.16) A

$\dfrac{1}{n!} - \dfrac{1}{(n+1)!}$

$= \dfrac{(n+1)! - n!}{n!(n+1)!} = \dfrac{(n+1)n! - n!}{n!(n+1)!}$

$= \dfrac{\cancel{n!}\ [n+1-1]}{\cancel{n!}\ (n+1)!} = \dfrac{n}{(n+1)!}$

7.17) C

$a_n = \dfrac{n!(n^2-1)}{(n+1)!} = \dfrac{\cancel{n!}\ \cancel{(n+1)}\ (n-1)}{\cancel{(n+1)}\ \cancel{n!}}$

$a_n = n - 1$

$a_{1984} = 1984 - 1 = 1983$

7.18) A

$\dfrac{(n-2)!}{1\,.\,2\,.\,3\,.\,...\,.\,(n-2)} = \dfrac{\cancel{(n-2)!}}{\cancel{(n-2)!}} = 1$

7.19) C

$2\,.\,4\,.\,6\,.\,8\,.\,10\,.\,...\,.\,(2n)$

$= 1\,.\,2\,.\,2\,.\,2\,.\,3\,.\,2\,.\,4\,.\,2\,.\,5\,.\,2\,.\,...\,.\,(n\,.\,2)$

$= (1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,...\,.\,n)\ \underbrace{(2\,.\,2\,.\,2\,.\,...\,.\,2)}_{\text{n vezes}}$

$= n!\,.\,2^n$

Aula 7

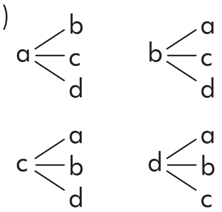
8.01) a) $A_{6,3} = \dfrac{6!}{(6-3)!} = \dfrac{6\,.\,5\,.\,4\,.\,\cancel{3!}}{\cancel{3!}} = 120$

b) $A_{10,4} = \dfrac{10!}{(10-4)!} = \dfrac{10\,.\,9\,.\,8\,.\,7\,.\,\cancel{6!}}{\cancel{6!}} = 5040$

c) $A_{20,1} = \dfrac{20!}{(20-1)!} = \dfrac{20\,.\,\cancel{19!}}{\cancel{19!}} = 20$

d) $A_{12,2} = \dfrac{12!}{(12-2)!} = \dfrac{12\,.\,11\,.\,\cancel{10!}}{\cancel{10!}} = 132$

8.02)



8.03) $A_{m,3} = 30\,m$

$$\frac{m!}{(m-3)!} = 30\,m$$

$$\frac{\cancel{m}\,.\,(m-1)\,(m-2)\,\cancel{(m-3)!}}{\cancel{(m-3)!}} = 30\,\cancel{m}$$

$m^2 - 2m - m + 2 = 30$
$m^2 - 3m - 28 = 0$
$m' = 7$
$\cancel{m'' = -4}$
$S = \{7\}$

8.04) a) $A_{5,2} = \frac{5!}{3!} = \frac{5\,.\,4\,.\,\cancel{3!}}{\cancel{3!}} = 20$

b) $A_{5,3} = \frac{5!}{2!} = \frac{5\,.\,4\,.\,3\,.\,\cancel{2!}}{\cancel{2!}} = 60$

c) $A_{5,4} = \frac{5!}{1!} = 5\,.\,4\,.\,3\,.\,2\,.\,1 = 120$

d) $A_{5,5} = \frac{5!}{0!} = 120$

8.05) {0, 1, ..., 9}

a) $9 \cdot 10 = 90$

b) $9 \cdot 9 = 81$

8.06) {1, 2, 3, 4, 5, 6}

$5 \cdot 4 \cdot 3 = 60$

8.07) {2, 4, $\cancel{6}$, 8, 9}

$\underline{2}$ $3 \cdot 2 = 6$

3 $\underline{2}$ $\cdot\, 2 = 6$

$3 \cdot 2 \cdot$ $\underline{2}$ $= 6$

Total: 18

8.08) {1, 2, 3, ..., 9}

$5 \cdot 8 \cdot 7 = 280$

8.09) C
{0, 1, 2, ..., 9}

$9 \cdot 9 \cdot 8 = 648$

8.10) A
{0, 1, 2, $\cancel{3}$, 4, 5, 6, 7, $\cancel{8}$, 9}
Maior do que 64000

$\underline{6}$ $4 \cdot 6 \cdot 5 \cdot 4 = 480$

$\underline{7}$ $7 \cdot 6 \cdot 5 \cdot 4 = 840$

$\underline{9}$ $7 \cdot 6 \cdot 5 \cdot 4 = 840$

Total: 2160

8.11) D
{1, 2, 3, 4, 5, 6}
Menores que 30000

$2\ \ 5\ \ 4\ \ 3\ \ 2 = 240$

8.12) $5 \cdot 4 \cdot 3 = 60$

8.13) $10 \cdot 9 \cdot 8 \cdot 7 = 5040$

8.14) a) $C_{5,2} = \frac{5!}{2!3!} = \frac{5\,.\,4\,.\,\cancel{3!}}{2\,.\,\cancel{3!}} = 10$

$C_{6,3} = \frac{6!}{3!3!} = \frac{\cancel{6}\,.\,5\,.\,4\,.\,\cancel{3!}}{\cancel{6}\,.\,\cancel{3!}} = 20$

$C_{5,2} + C_{6,3} = 30$

b) $C_{4,0} = \frac{\cancel{4!}}{0!\,\cancel{4!}} = 1$

$C_{4,1} = \frac{4!}{1!3!} = \frac{4\,.\,\cancel{3!}}{1!\,\cancel{3!}} = 4$

$C_{4,2} = \frac{4!}{2!2!} = \frac{4\,.\,3\,.\,\cancel{2!}}{2!\,\cancel{2!}} = 6$

$C_{4,3} = \frac{4!}{3!1!} = \frac{4\,.\,\cancel{3!}}{\cancel{3!}\,1!} = 4$

$C_{4,4} = \frac{4!}{4!0!} = 1$

$C_{4,0} + C_{4,1} + C_{4,2} + C_{4,3} + C_{4,4} = 16$

8.15) a) $\frac{n!}{1!(n-1)!} + \frac{n!}{2!(n-2)!} = 10$

$\frac{n\,.\,\cancel{(n-1)!}}{\cancel{(n-1)!}} + \frac{n\,.\,(n-1)\,\cancel{(n-2)!}}{2\,.\,\cancel{(n-2)!}} = 10$

$n + \frac{n^2 - n}{2} = 10$

$\frac{2n + n^2 - n}{\cancel{2}} = \frac{20}{\cancel{2}}$

$n^2 + n - 20 = 0$

$n' = 4$

$\cancel{n'' = -5}$

S = {4}

b) $\frac{n!}{(n-1)![n-(n-1)]!} = 10$

$\frac{n\,.\,\cancel{(n-1)!}}{\cancel{(n-1)!}\;1!} = 10$

n = 10

S = {10}

8.16) $\frac{n!}{5!\,(n-5)!} + \frac{n!}{6!\,(n-6)!} = \frac{(n+1)!}{6!\,(n-5)!}$

$\frac{\cancel{n!}}{\cancel{5!}\,(n-5)\,\cancel{(n-6)!}} + \frac{\cancel{n!}}{6\,.\,\cancel{5!}\,\cancel{(n-6)!}} = \frac{(n+1)\,\cancel{n!}}{6\,.\,\cancel{5!}\,(n-5)\,\cancel{(n-6)!}}$

$\frac{1}{n-5} + \frac{1}{6} = \frac{n+1}{6(n\,.\,5)} \Rightarrow \frac{6+n-5}{\cancel{6(n-5)}} = \frac{n+1}{\cancel{6(n-5)}}$

1 + n = 1 + n

Conclusão: A igualdade é válida para qualquer valor de n ≥ 6.

a) **Verdadeira**.

b) **Verdadeira**.

c) **Verdadeira**.

8.17) E

$2C_{n,\,4} - C_{n,\,3} = 0$

$2\,.\,\frac{\cancel{n!}}{4!(n-4)!} = \frac{\cancel{n!}}{3!(n-3)!}$

$\frac{\cancel{2}}{\cancel{24}\,.\,\cancel{(n-4)!}} = \frac{1}{6\,.\,(n-3)\,.\,\cancel{(n-4)!}}$

$\frac{1}{12} = \frac{1}{6(n-3)} \Rightarrow \cancel{6}(n-3) = \cancel{12}$

$n - 3 = 2 \Rightarrow n = 5$

8.18) A

$2A_{x,\,4} = 4!C_{x,\,x-5}$

$\cancel{2}\,.\,\frac{\cancel{x!}}{(x-4)!} = 4\,.\,3\,.\,\cancel{2}\,.\,1\,.\,\frac{\cancel{x!}}{(x-5)![x-(x-5)]!}$

$\frac{1}{(x-4)\;\cancel{(x-5)!}} = \frac{12}{\cancel{(x-5)!}\;5!}$

$12(x-4) = 5!$

$12(x-4) = 120 \Rightarrow x - 4 = 10 \Rightarrow x = 14$

8.19) a)

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBCA
ADBC	BDAC	CDAB	DCAB
ADCB	BDCA	CDBA	DCBA

b) ABCD

c) 24 vezes

8.20) a) $C_{6,6} = \frac{6!}{6!0!} = 1$

b) $C_{6,5} = \frac{6!}{5!1!} = \frac{6\,.\,\cancel{5!}}{\cancel{5!}\;1!} = 6$

c) $C_{6,4} = \frac{6!}{4!2!} = \frac{6\,.\,5\,.\,\cancel{4!}}{\cancel{4!}\;2!} = 15$

d) $C_{6,3} = \frac{6!}{3!3!} = \frac{\cancel{6}\,.5.4.\cancel{3!}}{\cancel{6}\,.\,\cancel{3!}} = 20$

e) $C_{6,2} = \frac{6!}{2!4!} = \frac{6\,.\,5\,.\,\cancel{4!}}{2\,.\,\cancel{4!}} = 15$

8.21) $C_{6,4} = \frac{6!}{4!2!} = \frac{6\,.\,5\,.\,\cancel{4!}}{\cancel{4!}\;2!} = 15$

8.22) A, ~~B~~, C, D, E, F

$C_{5,3} = \frac{5!}{3!2!} = \frac{5\,.\,4\,.\,\cancel{3!}}{\cancel{3!}\;2!} = 10$

8.23) $\{M_1, M_2, M_3, M_4, M_5, P_1, P_2, ..., P_{15}\}$

a) $C_{15,10} =$

$= \frac{15!}{10!5!} = \frac{\overset{3}{\cancel{15}}\,.\,\overset{7}{\cancel{14}}\,.\,13\,.\,\cancel{12}\,.\,11\,.\,\cancel{10!}}{\cancel{10!}\,.\,\cancel{5}\,.\,\cancel{4}\,.\,\cancel{3}\,.\,\cancel{2}\,.\,\cancel{1}} =$

$= 3003$

b) $C_{15,5} = \frac{15!}{5!10!} = 3003$

c) $C_{5,1}\,.\,C_{15,9} = \frac{5!}{1!\,4!}\,.\,\frac{15!}{9!6!} =$

$= \frac{5\,.\,\cancel{4!}}{1\,.\,\cancel{4!}}\,.\,\frac{\overset{3}{\cancel{15}}\,.\,\overset{7}{\cancel{14}}\,.\,13\,.\,\cancel{12}\,.\,11\,.\,\overset{5}{\cancel{10}}\,.\,\cancel{9!}}{\cancel{9!}\;\underset{2}{\cancel{6}}\,.\,\cancel{5}\,.\,\cancel{4}\,.\,\cancel{3}\,.\,\cancel{2}\,.\,1} =$

$= 5\,.\,5005 = 25025$

d) Com ou sem matemático:

$$C_{20,10} = \frac{20!}{10!10!} =$$

$$= \frac{\cancel{20}.19.\overset{2}{\cancel{18}}.17.\overset{2}{\cancel{16}}.\cancel{15}.\cancel{14}.13.\cancel{12}.11.\cancel{10!}}{\cancel{10}.\cancel{9}.\cancel{8}.\cancel{7}.\underset{2}{\cancel{6}}.\cancel{5}.\cancel{4}.\cancel{3}.\cancel{2}.1.\cancel{10!}} =$$

= 184756

Sem matemático:

$C_{15,10} = 3003$

Pelo menos um matemático:

184756 – 3003 = 181753

8.24) $M_1, M_2, ..., M_7, H_1, H_2, ..., H_6$

a) 3M e 2H

$$C_{7,3} \cdot C_{6,2} = \frac{7!}{3!4!} = \frac{6!}{2!4!}$$

$$= \frac{7.\cancel{6}.5.\cancel{4!}}{\cancel{6}.\cancel{4!}} \cdot \frac{\overset{3}{\cancel{6}}.5.\cancel{4}!}{\cancel{2}!\cancel{4}!}$$

= 35 . 15 = 525

b) Com nenhuma mulher:

$C_{6,5} = 6$

Com 1 mulher:

$C_{7,1} \cdot C_{6,4} = 7 . 15 = 105$

Com 2 mulheres:

$C_{7,2} \cdot C_{6,3} = 21 . 20 = 420$

Com 3 mulheres:

$C_{7,3} \cdot C_{6,2} = 35 . 15 = 525$

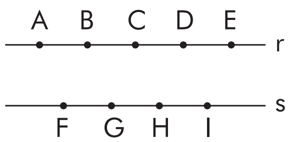
Com no máximo 3 mulheres:

6 + 105 + 420 + 525 = 1056

8.25) Verdadeira.

$$C_{20,3} = \frac{20!}{3!17!} = \frac{20.19.\overset{3}{\cancel{18}}.\cancel{17!}}{\cancel{3}.\cancel{2}.\cancel{1}.\cancel{17!}} = 1140$$

8.26)



a) $\underset{5}{\text{ponto em r}} \cdot \underset{4}{\text{ponto em s}} = 20$

b) $C_{9,3} - C_{5,3} - C_{4,3} = 84 - 10 - 4 = 70$

c) 2 vértices sobre **r** e 2 vértices sobre **s**:

$C_{5,2} \cdot C_{4,2} = 10 . 6 = 60$

8.27) D

$C_{8,3} - C_{3,3} - C_{5,3} = 56 - 1 - 10 = 45$

8.28) B

$$C_{10,3} = \frac{10!}{3!7!} = \frac{10.\overset{3}{\cancel{9}}.\overset{4}{\cancel{8}}.\cancel{7!}}{\cancel{3}.\cancel{2}.1.\cancel{7!}} = 120$$