Matemática E – Extensivo – V. 1

Resolva

Aula 1

1.01) P.A. $(x + 1, 2x, x^2 - 5)$

$$2x = \frac{(x+1) + (x^2 - 5)}{2}$$

$4x = x + x^2 - 4$
$0 = x^2 - 3x - 4$

$x' = 4$, $x'' = -1$

$x' = 4$ → P. A. (5, 8, 11)

$x'' = -1$ → Não faz sentido, pois os termos são lados de um triângulo.

Perímetro: 5 + 8 + 11 = 24

1.02) P.A. $(x - r, x, x + r)$

$x - \cancel{r} + x + x + \cancel{r} = 27$
$3x = 27$
$x = 9$
P.A. $(9 - r, 9, 9 + r)$

$(9 - r) \cdot \cancel{9} \cdot (9 + r) = \cancel{288}$
$(9 - r) \cdot (9 + r) = 32$
$81 - r^2 = 32$
$49 = r^2$
$r = \pm 7$
Se $r = 7 \Rightarrow$ P.A. (2, 9, 16)
Se $r = -7 \Rightarrow$ (16, 9, 2)

1.03) $\begin{cases} a_1 + a_7 = 92 \\ a_2 - a_1 = -5 \end{cases}$

$\begin{cases} a_1 + a_1 + 6r = 92 \\ \cancel{a_1} + r - \cancel{a_1} = -5 \end{cases}$

$\begin{cases} 2a_1 + 6r = 92 \\ r = -5 \end{cases}$

$2a_1 + 6 \cdot (-5) = 92$
$2a_1 = 122$
$a_1 = 61$

Aula 2

2.01) $a_n = 3n - 15$
$a_1 = 3 \cdot 1 - 15 = -12$
$a_2 = 3 \cdot 2 - 15 = -9$
$a_3 = 3 \cdot 3 - 15 = -6$
$a_4 = 3 \cdot 4 - 15 = -3$
$a_5 = 3 \cdot 5 - 15 = 0$
$a_6 = 3 \cdot 6 - 15 = 3$
$a_7 = 3 \cdot 7 - 15 = 6$
$a_8 = 3 \cdot 8 - 15 = 9$
$a_9 = 3 \cdot 9 - 15 = 12$
Soma = 0

2.02) $S_n = 2n(n + 2)$
$S_1 = 2 \cdot 1(1 + 2) = 6 = a_1$
$S_2 = 2 \cdot 2(2 + 2) = 16 = a_1 + a_2$
$a_2 = 10$
$r = 4$
P.A. (6, 10, 14, 18, 22)

2.03) C
P.A. (1, 2, 3, ...)
$S_n = 231$; $n = ?$
$a_n = 1 + (n - 1) \cdot 1$
$a_n = n$

$$S_n = \frac{(a_1 + a_n)n}{2}$$

$$231 = \frac{(1 + n) \cdot n}{2}$$

$462 = n^2 + n$
$0 = n^2 + n - 462$
$n' = 21$; $\cancel{n'' = -22}$

Aula 3

3.01) hora 0, hora 1, hora 2 → a_1, a_2, a_3

(8, 16, 32, 64, ...)
a_1 a_2 a_3

$a_{10} = a_1 \cdot q^9 = 16 \cdot 2^9 =$
$= 16 \cdot 512 = 8192$

3.02) P.G. $(1 + x, 3 + x, 2 + x)$
$(3 + x)^2 = (1 + x) \cdot (2 + x)$

$9 + 6x + \cancel{x^2} = 2 + x + 2x + \cancel{x^2}$
$3x = -7$

$$x = -\frac{7}{3}$$

3.03) 31
01. **Verdadeira.**
P.A. $(1 - 3x, x - 2, 2x + 1)$

$$x - 2 = \frac{1 - 3x + 2x + 1}{2}$$

$2x - 4 = -x + 2$
$3x = 6$
$x = 2$
P.A. (–5, 0, 5)

02. **Verdadeira.**
P.G. $(4y, 2y - 1, y + 1)$
$(2y - 1)^2 = 4y \cdot (y + 1)$
$\cancel{4y^2} - 4y + 1 = \cancel{4y^2} + 4y$
$1 = 8y \Rightarrow y = \frac{1}{8}$

04. **Verdadeira.**
P.A. $(-5, 0, +5)$
$-5 + 0 + 5 = 0$

08. **Verdadeira.**
P.G. $\left(\frac{1}{2}, \frac{1}{4} - 1, \frac{1}{8} + 1\right)$
P.G. $\left(\frac{1}{2}, -\frac{3}{4}, \frac{9}{8}\right)$
$q = \frac{-\frac{3}{4}}{\frac{1}{2}} = \frac{-3}{2}$

16. **Verdadeira.**
P.A. $(-5, 0, 5)$
$r = a2 - a1$
$r = 0 - (-5) \Rightarrow r = 5 > 0$
Portanto, a P.A. é crescente.

Aula 4

4.01) E

1	→ João	Soma: 511
2	→ 1ª etapa	
4	→ 2ª etapa	
8	→ 3ª etapa	
16	→ 4ª etapa	
32	→ 5ª etapa	
64	→ 6ª etapa	
128	→ 7ª etapa	
256	→ 8ª etapa	

4.02) $\left(1, \frac{1}{2}, \frac{1}{4}, \ldots\right)$

$S_\infty = \frac{a_1}{1 - q} = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2\ h$

Testes

Aula 1

1.01) D
(F) (2, 6, 10, 14, ...) é uma P.A. com r = 4.
(V) (3, 7, 11, 15, ...) $a_{1000} = 3 + 999 \cdot 4 = 3999$
(V) $\frac{(n+2)!}{3^n} = a_n$

$a_1 = \frac{3!}{3} = \frac{6}{3} = 2$

$a_2 = \frac{4!}{9} = \frac{24}{9} = \frac{8}{3}$

$a_3 = \frac{5!}{27} = \frac{120}{27} = \frac{40}{9}$

$a_4 = \frac{6!}{81} = \frac{720}{81} = \frac{80}{9}$

1.02) A
P.A. $\left(\frac{1}{2}, \frac{4}{2}, \frac{7}{2}, \ldots\right)$ $r = \frac{3}{2}$

$a_{24} = \frac{1}{2} + 23 \cdot \frac{3}{2} =$

$= \frac{1}{2} + \frac{69}{2} = \frac{70}{2} = 35$

1.03) C
$a_{31} = 2 + 30 \cdot 3$
$a_{31} = 92$

1.04) $a_8 = 30$
$a_9 = 35$
$r = 5$
$a_8 = a_1 + 7r$
$30 = a_1 + 7 \cdot 5$
$a_1 = -5$

1.05) C
$(3m, m + 1, 5)$
$m + 1 = \frac{3m + 5}{2}$
$2m + 2 = 3m + 5$
$-3 = m$
P.A. $(-9, -2, 5)$
$r = 7$

1.06) B
P.A. $(r - 1, 3r - 1, r - 3)$
$3r - 1 = \frac{r - 1 + r - 3}{2}$
$6r - 2 = 2r - 4$
$4r = -2$
$r = \frac{-1}{2}$

1.07) A

P.A. $(x - r, x, x + r)$

$x - \cancel{r} + x + x + \cancel{r} = 15$

$3x = 15$

$x = 5$

P.A. $(5 - r, 5, 5 + r)$

Olhando para a idade do mais moço $(5 - r)$, percebemos que o maior valor inteiro de **r** é 5.

Assim, o irmão mais velho pode ter $5 + 5 = 10$.

1.08) D

(0, 1, 4, 9, 16, 25, ...)

Diferenças entre os consecutivos: (1, 3, 5, 7, 9, ...)

Formam uma P.A.

1.09) P.A. $(x - r, x, x + r)$

$x - r + x + x + r = 15$

$3x = 15$

$x = 5$

P.A. $(5 - r, 5, 5 + r)$

$(5 - r) \cdot \cancel{5} \cdot (5 + r) = \cancel{105}$

$(5 - r) \cdot (5 + r) = 21$

$25 - r^2 = 21$

$4 = r^2$

$r = \pm 2$

$r = 2 \Rightarrow$ P.A. (3, 5, 7)

$r = -2 \Rightarrow$ P.A. (7, 5, 3)

Maior: 7

1.10) E

P.A. $(x - r, x, x + r)$

$x - \cancel{r} + x + x + \cancel{r} = 180°$

$3x = 180°$

$x = 60°$

P.A. $(60 - r, 60, 60 + r)$

$60 + r = 2 \cdot (60 - r)$

$60 + r = 120 - 2r$

$3r = 60$

$r = 20°$

P.A. (40°, 60°, 80°)

Menor: 40°

1.11) B

P.A. (11, __, __, __, __, __, __, __, 43)

$a_n = a_1 + (n - 1) \cdot r$

$43 = 11 + (9 - 1) \cdot r$

$32 = 8r$

$r = 4$

1.12) C

P.A. (104, 112, 120, 128, ..., 992)

$a_n = a_1 + (n - 1) \cdot r$

$992 = 104 + (n - 1) \cdot 8$

$992 = 104 + 8n - 8$

$n = 112$

$$S_n = \frac{(a_1 + a_n) \cdot n}{2} = \frac{(104 + 992) \cdot 112}{2} = 61376$$

1.13) P.A. (14, 21, ..., 994)

$a_n = a_1 + (n - 1) \cdot r$

$994 = 14 + (n - 1) \cdot 7$

$994 = 14 + 7n - 7$

$987 = 7n$

$n = 141$

1.14) $a_n = a_1 + (n - 1) \cdot r$

$12 = a_1 + (10 - 1) \cdot (-5)$

$12 = a_1 - 45$

$57 = a_1$

1.15) P.A. $(x - 3r, x - r, x + r, x + 3r)$

$x - \cancel{3r} + x - \cancel{r} + x + \cancel{r} + x + \cancel{3r} = -6$

$4x = -6$

$$x = \frac{-3}{2}$$

$$\text{P.A.} \left(-\frac{3}{2} - 3r, -\frac{3}{2} - r, -\frac{3}{2} + r, -\frac{3}{2} + 3r\right)$$

$$\left(-\frac{3}{2} - 3r\right) \cdot \left(-\frac{3}{2} + 3r\right) = -54$$

$$\frac{9}{4} - 9r^2 = -54$$

$$\frac{9}{4} + 54 = 9r^2$$

$$\frac{9 + 216}{4} = 9r^2$$

$$\frac{\cancel{225}}{4} = \cancel{9}r^2$$

$$\frac{25}{4} = r^2$$

$$r = \pm \frac{5}{2}$$

$$r = \frac{5}{2} \Rightarrow \text{P.A. } (-9, -4, 1, 6)$$

$$r = -\frac{5}{2} \Rightarrow \text{P.A. } (6, 1, -4, -9)$$

1.16) E

P.A. (–72, –65, –58, ...)

$r = 7$

$a_n > 0$

$a_n \; -72 + (n - 1) \cdot 7$

$a_n = 7n - 79$

$7n - 79 > 0$

$$n > \frac{79}{7}$$

$n > 11{,}28$

$n = 12$

$a_{12} = -72 + (12 - 1) \cdot 7$

$a_{12} = -72 + 77$

$a_{12} = 5$

1.17) $a_1 = 6; a_5 = ?$

$a_8 + a_{10} = 10$
$a_1 + 7r + a_1 + 9r = 10$
$6 + 7r + 6 + 9r = 10$
$16r = -2$

$r = -\frac{1}{8}$

$a_5 = a_1 + 4r =$

$= 6 + 4 \cdot \left(-\frac{1}{8}\right) = 6 - \frac{1}{2} = \frac{11}{2}$

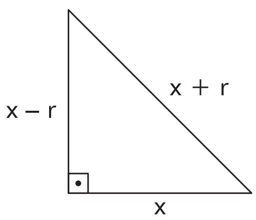
1.18) $a_7 = 20; a_{10} = 38; a_{20} = ?$

$a_{10} = a_7 + 3r$
$38 = 20 + 3r$
$18 = 3r$
$r = 6$
$a_{20} = a_{10} + 10r$
$a_{20} = 38 + 10 \cdot 6$
$a_{20} = 98$

1.19) P.A. $(x - 2r, x - r, x, x + r, x + 2r)$

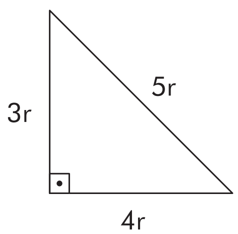
$x - \cancel{2r} + x - \cancel{r} + x + x + \cancel{r} + x + \cancel{2r} = 540°$
$5x = 540°$
$x = 108°$

1.20) P.A. $(x - r, x, x + r)$



$(x + r)^2 = (x - r)^2 + x^2$

$\cancel{x^2} + 2xr + \cancel{r^2} = \cancel{x^2} - 2xr + \cancel{r^2} + x^2$
$0 = x^2 - 4xr \qquad \div (x)$
$0 = x - 4r$
$x = 4r$
P.A. $(3r, 4r, 5r)$



$S = \frac{\overset{2}{\cancel{4}} r \cdot 3r}{\cancel{2}} = 150$

$6r^2 = 150$
$r^2 = 25$
$r = 5$
P.A. $(15, 20, 25)$

1.21) D

P.A. $(4, 7, 10, ...)$
$r = 3$
$a_n = 31; n = ?$
$a_n = a_1 + (n - 1) \cdot r$
$31 = 4 + (n - 1) \cdot 3$
$31 = 4 + 3n - 3$
$30 = 3n$
$n = 10$

1.22) C

$n = 7$

$\begin{cases} a_1 + a_2 = 14 \\ a_6 + a_7 = 54 \end{cases}$

$\begin{cases} a_1 + a_1 + r = 14 \\ a_1 + 5r + a_1 + 6r = 54 \end{cases}$

$\begin{cases} 2a_1 + r = 14 \quad . (-1) \\ 2a_1 + 11r = 54 \end{cases}$

$\begin{cases} -\cancel{2a_1} - r = -14 \\ \cancel{2a_1} + 11r = 54 \end{cases} \oplus$

$10r = 40$
$r = 4$
$2a_1 + r = 14$
$2a_1 + 4 = 14$
$a_1 = 5$
$a_7 = a_1 + 6r =$
$= 5 + 6 \cdot 4 = 29$

1.23) D

$a_n = 3n + 2$
$(5, 8, 11, 14, 17)$
a_1, a_2, a_3, a_4, a_5
$5 + 11 + 17 = 33$

1.24) a) Múltiplos de 9: (108, 117, ..., 999)

$a_n = a_1 + (n - 1) \cdot r$
$999 = 108 + (n - 1) \cdot 9$
$999 = 108 + 9n - 9$
$900 = 9n$
$n = 100$

b) Múltiplos de 15: (105, 120, ..., 990)

$990 = 105 + (n - 1) \cdot 15$
$990 = 105 + 15n - 15$
$900 = 15n$
$n = 60$

Múltiplos de 9 e 15 são os múltiplos do m.m.c. (9, 15) = 45
P.A. (135, 180, 225, ..., 990)
$990 = 135 + (n - 1) \cdot 45$
$990 = 135 + 45n - 45$
$20 = n$
Logo, o total de múltiplos de 9 ou 15 será:
$100 + 60 - 20 = 140$

Aula 2

2.01) E

$a_n = 3n + 2$
$a_1 = 3 . 1 + 2 = 5$
$a_{20} = 3 . 20 + 2 = 62$

$$S_{20} = \frac{(a_1 + a_{20}) . 20}{2} = \frac{(5 + 62) . 20}{2} = 670$$

2.02) D

$a_3 = 11; r = 4; S_{20} = ?$
$a_{20} = a_3 + 17r$
$= 11 + 17 . 4 = 79$
$a_3 = a_1 + 2r$
$11 = a_1 + 2 . 4$
$3 = a_1$

$$S_{20} = \frac{(a_1 + a_{20}) . 20}{2} = \frac{(3 + 79) . 20}{2} = 820$$

2.03) P.A. (1, 3, 5, 7, 9, 11, 13, 15, 17, 19)

$$S_{10} = \frac{(a_1 + a_{10}) . 10}{2} = \frac{(1 + 19) . 10}{2} = 100$$

2.04) P.A. (1, 3, 5, 7, ..., n)

$a_n = a_1 + (n - 1) . r = 1 + (n - 1) . 2 = 2n - 1$

$$S_n = \frac{(a_1 + a_n) . n}{2} = \frac{(\not{1} + 2n - \not{1}) . n}{2} = \frac{2n^2}{2} = n^2$$

2.05)

		Soma
3 4 5	⟶	12
6 8 10	⟶	24
9 12 15	⟶	36
12 16 20	⟶	48

Basta somar os 20 primeiros termos da P.A. (12, 24, 36, 48, ...)

$a_{20} = 12 + 19 . 12 = 240$

$$S_{20} = \frac{(12 + 240) . 20}{2} = 2520$$

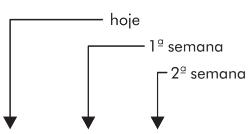
2.06) P.A. (202, 206, 210, ...); r = 4

$a_{35} = 202 + 34 . 4 = 338$
$a_{50} = 202 + 49 . 4 = 398$

$$S_{50} = \frac{(202 + 398) . 50}{2} = 15000$$

$S_{50} - a_{35} = 15000 - 338 = 14662$

2.07) C



P. A. (120; 119,5; 119; ...)

$r = -0,5; a_n = 80$ kg; $n = ?$
$a_1 = 119,5$
$a_n = a_1 + (n - 1) . r$
$80 = 119,5 + (n - 1) . (-0,5)$
$80 = 119,5 - 0,5n + 0,5$
$0,5n = 40$
$n = 80$

De outra forma:
Como emagrece 500 g por semana e precisa perder 40 kg, levará:
(500 g) . n = 40 Kg
$0,5n = 40$
$n = 80$

2.08) D

$S_n = n(n + 1)$
$a_{20} = S_{20} - S_{19} =$
$= 20(20 + 1) - 19(19 + 1) =$
$= 420 - 380 = 40$

2.09) $S_n = n^2$

$S_1 = 1 = a_1$
$S_2 = 4 = a_1 + a_2$
$a_2 = 3$
P.A. (1, 3, 5, 7, 9, ...)

2.10) $S_n = n^2 + 2n$

$S_1 = 1 + 2 = 3 = a_1$
$S_2 = 4 + 4 = 8 = a_1 + a_2$
$a_2 = 5$
P.A. (3, 5, 7, 9, ...)
$r = 2$

2.11) P.A. (–133, –126, –119, –112, ...)

$r = 7$
$a_n = a_1 + (n - 1) . r =$
$= -133 + (n - 1) . 7 =$
$= 7n - 140$
$S_n > 0$

$$S_n = \frac{(a_1 + a_n) . n}{2} =$$

$$= \frac{(-133 + 7n - 140) . n}{2} =$$

$$= \frac{(7n - 273) . n}{2}$$

Vamos verificar se existe algum **n** para o qual $S_n = 0$.

$$\frac{(7n - 273) . n}{2} = 0$$

$(7n - 273) . n = 0$
$n = 0$
ou
$7n - 273 = 0$
$n = 39$

Logo, o número mínimo de termos que torna $S_n > 0$ é $n = 40$.

2.12) E

P.A. (–7, –3, ...); r = 4

$a_n = -7 + (n - 1) \cdot 4$

$a_n = 4n - 11$

$$S_n = \frac{(-7 + 4n - 11) \cdot n}{2} = 3150$$

$(4n - 18) \cdot n = 6300$

$4n^2 - 18n - 6300 = 0 \quad \div (2)$

$2n^2 - 9n - 3150 = 0$

$n' = 42$

~~n" = –37,5~~

2.13) D

P.A. (20, 24, 28, 32, ...)

$a_n = 20 + (n - 1) \cdot 4$

$a_n = 4n + 16$

$$S_n = \frac{(20 + 4n + 16) \cdot n}{2} = 800$$

$(4n + 36) \cdot n = 1600$

$4n^2 + 36n - 1600 = 0 \quad \div (4)$

$n^2 + 9n - 400 = 0$

$n' = 16$

~~n" = –25~~

2.14) 04

P.A. (10, 20, 30, ..., 1990)

$a_1 = 10;\ a_n = 1990;\ n = 199$

$$S_n = \frac{(10 + 1990) \cdot 199}{2} = 199000$$

2.15) D

$S_n = n^2 + 4n$

$S_1 = 1 + 4 = 5 = a_1$

$S_2 = 4 + 8 = 12 = a_1 + a_2$

$a_2 = 7$

P.A. (5, 7, 9, 11, ...)

$a_n = a_1 + (n - 1) \cdot r$

$a_n = 5 + (n - 1) \cdot 2$

$a_n = 2n + 3$

2.16) $a_1 + a_2 + ... + a_{60} = 525$

$$\frac{(a_1 + a_{60}) \cdot \cancel{60}^{30}}{\cancel{2}} = 525$$

$(a_1 + a_{60}) \cdot \cancel{30} = \cancel{525}$

$a_1 + a_1 + 59r = 17,5$

$a_{61} + a_{62} + ... + a_{120} = 4125$

$$\frac{(a_{61} + a_{120}) \cdot \cancel{60}}{\cancel{2}} = 4125$$

$(a_{61} + a_{120}) \cdot 30 = 4125$

$a_1 + 60r + a_1 + 119r = 137,5$

$$\begin{cases} 2a_1 + 59r = 17,5 & \cdot(-1) \\ 2a_1 + 179r = 137,5 \end{cases}$$

$$\begin{cases} -2a_1 - 59r = -17,5 \\ 2a_1 + 179r = 137,5 \end{cases} \oplus$$

$120r = 120$

$r = 1$

$2a_1 + 59r = 17,5$

$2a_1 + 59 = 17,5$

$2a_1 = -41,5$

$$2a_1 = \frac{-83}{2}$$

$$a_1 = \frac{-83}{4}$$

$$\text{P.A.} \left(\frac{-83}{4}, \frac{-79}{4}, \frac{-75}{4}, ... \right)$$

2.17) $a_1 = 3;\ a_n = 31;\ S_n = 136;\ n = ?$

$$S_n = \frac{(a_1 + a_n) \cdot n}{2}$$

$$136 = \frac{(3 + 31) \cdot n}{2}$$

$$136 = \frac{\cancel{34}n}{2}$$

$136 = 17 \cdot n$

$n = 8$

2.18) $n = 9;\ r = 2;\ S_n = 0;\ a_6 = ?$

$a_9 = a_1 + 8r$

$a_9 = a_1 + 16$

$$S_9 = \frac{(a_1 + a_9) \cdot 9}{2}$$

$$0 = \frac{(a_1 + a_9) \cdot 9}{2}$$

$a_1 + a_9 = 0$

$a_1 + (a_1 + 16) = 0$

$2a_1 = -16$

$a_1 = -8$

$a_6 = a_1 + 5r = -8 + 10 = 2$

2.19) $S_{21} = ?$

$$\begin{cases} a_3 + a_7 = 100 \\ a_6 + a_9 = 250 \end{cases}$$

$$\begin{cases} a_1 + 2r + a_1 + 6r = 100 \\ a_1 + 5r + a_1 + 8r = 250 \end{cases}$$

$$\begin{cases} 2a_1 + 8r = 100 & \cdot(-1) \\ 2a_1 + 13r = 250 \end{cases}$$

$$\begin{cases} -\cancel{2a_1} - 8r = -100 \\ \cancel{2a_1} + 13r = 250 \end{cases} \oplus$$

$5r = 150$

$r = 30$

$a_1 = -70$

$a_{21} = a_1 + 20r = -70 + 600 = 530$

$$S_{21} = \frac{(a_1 + a_{21}) \cdot 21}{2} = \frac{(-70 + 530) \cdot 21}{2} = 4830$$

2.20) $\begin{cases} a_1 + a_2 = 3 \\ a_1 + a_2 + a_3 = 2 \end{cases}$

$\begin{cases} a_1 + a_1 + r = 3 \\ a_1 + a_1 + r + a_1 + 2r = 2 \end{cases}$

$\begin{cases} 2a_1 + r = 3 \quad .(-3) \\ 3a_1 + 3r = 2 \end{cases}$

$\begin{cases} -6a_1 - \cancel{3r} = -9 \\ 3a_1 + \cancel{3r} = 2 \end{cases} \oplus$

$-3a_1 = -7$

$a_1 = \frac{7}{3}$

$2a_1 + r = 3$

$\frac{14}{3} + r = 3$

$r = 3 - \frac{14}{3}$

$r = \frac{-5}{3}$

P.A. $\left(\frac{7}{3}, \frac{2}{3}, \frac{-3}{3}, \frac{-8}{3}, \frac{-13}{3}\right)$

Soma $= \frac{7}{3} + \frac{2}{3} - \frac{3}{3} - \frac{8}{3} - \frac{13}{3} = \frac{-15}{3} = -5$

2.21) B

P.A. (1, 2, 3, 4, ...); r = 1

$a_1 = 1$; $a_n = n$; n = ?

$S_n = 171$

$S_n = \frac{(a_1 + a_n) \cdot n}{2}$

$171 = \frac{(1 + n) \cdot n}{2}$

$342 = n^2 + n$

$0 = n^2 + n - 342$

n' = 18

~~n" = –19~~

Aula 3

3.01) P.G. $\left(10, 2, \frac{2}{5}, \frac{2}{25}, ...\right)$

$q = \frac{2}{10} = \frac{1}{5}$

Assim:

$a_5 = \frac{2}{25} \cdot \frac{1}{5} = \frac{2}{125}$

$a_6 = \frac{2}{125} \cdot \frac{1}{5} = \frac{2}{625}$

Posição 6

3.02) P.G. (a – 2 + x, a + x, a + 3 + x)

$(a + x)^2 = (a - 2 + x) \cdot (a + 3 + x)$

$\cancel{a^2} + 2ax + \cancel{x^2} = \cancel{a^2} + 3a + ax - 2a - 6 - 2x + ax +$

$+ 3x + \cancel{x^2}$

$\cancel{2ax} = a + \cancel{2ax} + x - 6$

$0 = a + x - 6$

$x = 6 - a$

3.03) A

P.G. (4x, 2x + 1, x – 1)

$(2x + 1)^2 = 4x(x - 1)$

$\cancel{4x^2} + 4x + 1 = \cancel{4x^2} - 4x$

$8x = -1$

$x = \frac{-1}{8}$

3.04) B

P.G. $\left(\frac{1}{9}, \frac{1}{3}, 1, ..., 729\right)$

$q = 3$

$a_n = a_1 \cdot q^{n-1}$

$729 = \frac{1}{9} \cdot 3^{n-1}$

$3^6 = \frac{1}{3^2} \cdot 3^{n-1}$

$3^6 \cdot 3^2 = 3^{n-1}$

$\cancel{3}^8 = \cancel{3}^{n-1}$

$8 = n - 1$

$9 = n$

3.05) $(13 + x)^2 = (1 + x) \cdot (49 + x)$

$169 + 26x + \cancel{x^2} = 49 + x + 49x + \cancel{x^2}$

$120 = 24x$

$x = 5$

3.06) $a_9 = a_5 \cdot q^4$

$a_9 = 3 \cdot (\sqrt{2})^4 = 3 \cdot 2^2 = 12$

3.07) P.G. (–1 + x, 1 + x, 4 + x)

$(1 + x)^2 = (-1 + x) \cdot (4 + x)$

$1 + 2x + \cancel{x^2} = -4 - x + 4x + \cancel{x^2}$

$5 = x$

3.08) D

(a_5, a_6, a_7)

$(10, a_6, 16)$

$a_6^2 = 10 \cdot 16$

$a_6 = \sqrt{10 \cdot 16} = 4\sqrt{10}$

3.09) D

$a_6 = a_3 \cdot q^3$

$80 = 10 \cdot q^3$

$8 = q^3$

$q = 2$

3.10) C

P.G. $(x, 8x, 64x)$

base ↙ ↓ altura ↘ área

$$\text{área} = \frac{b \cdot h}{2}$$

$$64x = \frac{x \cdot 8x}{2}$$

$64\not{x} = 4x^{\not{2}}$

$64 = 4x$

$x = 16$

3.11) B

P.G. $(2q, 2q^2, 2q^3, \ldots)$

$\quad a_1 \quad a_2 \quad a_3$

$a_1 + a_2 = 24$

$2q + 2q^2 = 24$

$q + q^2 = 12$

$q^2 + q - 12 = 0$

$q' = -4$

$q'' = 3$

3.12) $q = 5$

$(a_1, a_2, a_3, a_4, a_5, a_6)$

$a_1 \cdot a_6 = 12500$

$a_1 \cdot a_1 \cdot q^5 = 12500$

$a_1^2 \cdot 5^5 = 12500$

$a_1^2 \cdot 3125 = 12500$

$a_1^2 = 4$

$a_1 = 2$

$a_3 = a_1 \cdot q^2 =$

$= 2 \cdot 5^2 = 50$

3.13) $q = ?$

$$\begin{cases} a_1 + a_2 = 1 \\ a_3 + a_4 = 9 \end{cases}$$

$$\begin{cases} a_1 + a_1q = 1 \\ a_1q^2 + a_1q^3 = 9 \end{cases}$$

$$\begin{cases} \not{a_1} \cdot \not{(1+q)} = 1 \\ \not{a_1}q^2 \cdot \not{(1+q)} = 9 \end{cases} \quad (\div)$$

$$\frac{1}{q^2} = \frac{1}{9}$$

$q = 3$

3.14) A

Ano 1: $a_1 = V$

Ano 2: $a_2 = V - 30\%V = 70\%V$

Ano 3: $a_3 = 70\%V - 30\%\,(70\%V) =$

$= 70\%V\,(1 - 30\%) =$

$= 70\%V \cdot 70\% =$

$= (70\%)^2V$

P.G. $(V, 70\%V, (70\%)^2, V, \ldots)$

$q = 70\%$

$a_8 = a_1 \cdot q^7 = V \cdot (70\%)^7 =$

$= (0{,}7)^7 \cdot V$

3.15) C

1º mês: x

2º mês: $x + 0{,}2x = 1{,}2x$

3º mês: $1{,}2x + 0{,}2 \cdot (1{,}2x) =$

$= (1{,}2x) \cdot (1 + 0{,}2) =$

$= 1{,}2x \cdot 1{,}2 = (1{,}2)^2\,x$

P.G. $(x;\ 1{,}2x;\ (1{,}2)^2x;\ \ldots)$

$q = 1{,}2$

3.16) A

P.G. (4, __, __, __, __, __, 2916)

$a_7 = a_1q^6$

$2916 = 4 \cdot q^6$

$729 = q^6$

$q = \pm\sqrt[6]{729}$

$q = \pm 3$

3.17) P.G. (8, __, __, __, __, __, 5832)

$a_7 = a_1 \cdot q^6$

$5832 = 8 \cdot q^6$

$729 = q^6$

$q = \pm\sqrt[6]{729}$

$q = \pm 3$

$a_5 = a_1 \cdot q^4 =$

$= 8 \cdot (\pm 3)^4 = 8 \cdot 81 = 648$

3.18) A

$$\begin{cases} a_1 + a_2 = 32 \\ a_4 + a_5 = 864 \end{cases}$$

$$\begin{cases} a_1 + a_1q = 32 \\ a_1q^3 + a_1q^4 = 864 \end{cases}$$

$$\begin{cases} \not{a_1} \cdot \not{(1+q)} = 32 \\ \not{a_1}q^3 \cdot \not{(1+q)} = 864 \end{cases} \div$$

$$\frac{1}{q^3} = \frac{1}{27}$$

$q = 3$

$a_1 = 8$

P.G. $(8, 24, 72, \ldots)$

$a_3 = 72$

3.19) $\begin{cases} a_6 + a_7 + a_8 = 224 \\ a_4 + a_5 + a_6 = 56 \end{cases}$

$$\begin{cases} a_1q^5 + a_1q^6 + a_1q^7 = 224 \\ a_1q^3 + a_1q^4 + a_1q^5 = 56 \end{cases}$$

$$\begin{cases} a_1q^5 \cdot \cancel{(1+q+q^2)} = 224 \\ a_1q^3 \cdot \cancel{(1+q+q^2)} = 56 \end{cases} \quad (\div)$$

$q^2 = 4$
$q = 2$
$a_1q^3 + a_1q^4 + a_1q^5 = 56$
$8a_1 + 16a_1 + 32a_1 = 56$
$56a_1 = 56$
$a_1 = 1$
P.G. (1, 2, 4, 8, ...)
$a_3 = 4$

3.20) $\begin{cases} a_2 + a_4 = 10 \\ a_1 + a_3 = 5 \end{cases}$

$$\begin{cases} a_1q + a_1q^3 = 10 \\ a_1 + a_1q^2 = 5 \end{cases}$$

$$\begin{cases} \cancel{a_1}q \cdot \cancel{(1+q^2)} = 10 \\ \cancel{a_1} \cdot \cancel{(1+q^2)} = 5 \end{cases} \quad (\div)$$

$q = 2$
$a_1 = 1$
P.G. (1, 2, 4, 8)

3.21) $q = \dfrac{2}{3}$

P.G. $\left(x, \dfrac{2x}{3}, \dfrac{4x}{9}\right)$

$x + \dfrac{2x}{3} + \dfrac{4x}{9} = 57$

$\dfrac{9x + 6x + 4x}{9} = 57$

$\dfrac{\cancel{19}x}{9} = \cancel{57}^{3}$

$x = 27$

Termo médio: $\dfrac{2x}{3} = \dfrac{2}{3} \cdot 27 = 18$

Aula 4

4.01) a) $S = 1 + 2 + 4 + ... + 4096$

$$S = \frac{a_nq - a_1}{q - 1} = \frac{4096 \cdot 2 - 1}{2 - 1} = 8191$$

b) $S = 2 + 1 + \dfrac{1}{2} + ... + \dfrac{1}{1024}$

$$S = \frac{a_n \cdot q - a_1}{q - 1} = \frac{\frac{1}{1024} \cdot \frac{1}{2} - 2}{\frac{1}{2} - 1} = \frac{\frac{1}{2048} - 2}{-\frac{1}{2}} =$$

$$= \frac{\frac{-4095}{2048}}{-\frac{1}{2}} = \frac{4095 \cdot \cancel{2}}{\cancel{2048}} = \frac{4095}{1024}$$

c) $S = 2 + 1 + \dfrac{1}{2} + \dfrac{1}{4} + ...$

$$S_\infty = \frac{a_1}{1 - q} = \frac{2}{1 - \frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$$

d) $S = 18 + 12 + 8 + ...$

$$q = \frac{12}{18} = \frac{2}{3}$$

$$S_\infty = \frac{a_1}{1 - q} = \frac{18}{1 - \frac{2}{3}} = \frac{18}{\frac{1}{3}} = 18 \cdot 3 = 54$$

4.02) 18

01. **Incorreta.**

P.G. $\left(1, \dfrac{1}{2}, \dfrac{1}{4}, ...\right)$

$$S_\infty = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

02. **Correta.**

$(a_1, a_2, a_3, a_4, a_5, a_6, a_7)$

$a_4^2 = a_1 \cdot a_7$

$a_4^2 = \dfrac{1}{32}$

$a_4^2 = 2^{-5}$

$a_4 = \sqrt{2^{-5}} = 2^{-\frac{5}{2}}$

04. **Incorreta.**

$x^2 = (x - 2) \cdot (x + 4)$

$\cancel{x^2} = \cancel{x^2} + 4x - 2x - 8$
$8 = 2x$
$x = 4$
P.G. (2, 4, 8)
$q = 2$

08. **Incorreta.**

$$S_n = \frac{a_n \cdot q - a_1}{q - 1} = \frac{486 \cdot 3 - 2}{3 - 1} = 728$$

16. **Correta.**

$$S_\infty = \frac{x}{1 - \frac{1}{3}} = 60$$

$$\frac{x}{\frac{2}{3}} = 6$$

$x = 60 \cdot \dfrac{2}{3}$

$x = 40$

4.03) B

$$\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + ... + \frac{2}{3^n}$$

$q = \frac{1}{3}$

$S_n = \frac{a_n q - a_1}{q - 1} =$

$= \frac{\frac{2}{3^n} \cdot \frac{1}{3} - \frac{2}{3}}{\frac{1}{3} - 1} = \frac{\frac{2}{3 \cdot 3^n} - \frac{2}{3}}{\frac{-2}{3}} = \frac{\frac{2}{3}\left[\frac{1}{3^n} - 1\right]}{\frac{-2}{3}} =$

$= -\left[\frac{1}{3^n} - 1\right] = 1 - \frac{1}{3^n} = 1 - 3^{-n}$

4.04) E

$S_n = \frac{a_1 \cdot (q^n - 1)}{q - 1} = \frac{2 \cdot (2^{500} - 1)}{2 - 1} = 2(2^{500} - 1)$

4.05) B

$S_\infty = \frac{a_1}{1 - q}$

$2 = \frac{x}{1 - x}$

$2 - 2x = x$
$2 = 3x$

$x = \frac{2}{3}$

4.06) 14

01. **Incorreta.**
É uma P.A. com r = 2.

02. **Correta.**
(1, 3, 5, 7, ...)
$a_{1000} = a_1 + 999 \cdot r =$
$= 1 + 999 \cdot 2 = 1999$

04. **Correta.**

$a_n = \frac{(n+2)!}{3^n}$

$a_1 = \frac{3!}{3} = \frac{6}{3} = 2$

$a_2 = \frac{4!}{9} = \frac{24}{9} = \frac{8}{3}$

$a_3 = \frac{5!}{27} = \frac{120}{27} = \frac{40}{9}$

$a_4 = \frac{6!}{81} = \frac{720}{81} = \frac{80}{9}$

08. **Correta.**

$S_\infty = \frac{a_1}{1 - q}$

$50 = \frac{\frac{x}{2}}{1 - \frac{1}{2}}$

$50 = \frac{\frac{x}{\cancel{2}}}{\frac{1}{\cancel{2}}}$

$x = 50$

4.07) $S = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{9} + \frac{1}{8} + \frac{1}{27} + \ldots$

$S' = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

$S'' = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots = \frac{\frac{1}{3}}{1 - \frac{1}{3}} + \frac{\frac{1}{3}}{\frac{2}{3}} + \frac{1}{2}$

Logo, $S = S' + S'' = 1 + \frac{1}{2} = \frac{3}{2}$

4.08) $x + \frac{x}{a} + \frac{x}{a^2} + \frac{x}{a^3} + \ldots = \frac{3a}{a - 1}$

$S_\infty = \frac{x}{1 - \frac{1}{a}} = \frac{x}{\frac{a - 1}{a}} = x \cdot \frac{a}{a - 1}$

$\frac{x\cancel{a}}{\cancel{a - 1}} = \frac{3\cancel{a}}{\cancel{a - 1}}$

$x = 3$

4.09) a) Tábuas
(1, 1, 2, 4, 8, 16, 32, 64, 128)
Total
1 + 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 =
256 tábuas

b) Altura
256 . 0,5 cm = 128 cm = 1,28 m

4.10) D

$\frac{1}{10^2} + \frac{1}{10^4} + \frac{1}{10^6} + \ldots$

$S_\infty = \frac{\frac{1}{100}}{1 - \frac{1}{100}} = \frac{\frac{1}{100}}{\frac{99}{100}} = \frac{1}{99}$

4.11) B

$a_n = \frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots + \frac{n}{n^2} = \frac{1 + 2 + 3 + \ldots + n}{n^2}$ (I)

1 + 2 + 3 + ... + n é a soma dos termos de uma P.A. Usando a fórmula $S_n = \frac{(a_1 + a_n) \cdot n}{2}$, temos:

$$1 + 2 + 3 + \dots + n = \frac{(1+n) \cdot n}{2}$$

Voltando a I, obtemos:

$$a_n = \frac{\frac{(1+n) \cdot n}{2}}{n^2} = \frac{(1+n) \cdot \cancel{n}}{2} \cdot \frac{1}{\cancel{n}^2}$$

$$a_n = \frac{1+n}{2n} = \frac{1}{2n} + \frac{\cancel{n}}{2\cancel{n}}$$

$$a_n = \frac{1}{2n} + \frac{1}{2}$$

Quando **n** cresce, $\frac{1}{2n}$ tende a zero. Com isso,

$$a_n = \frac{1}{2n} + \frac{1}{2} \text{ tende a } \frac{1}{2}.$$

4.12) 02

01. **Incorreta.**

$$a_5^2 = a_3 \cdot a_7$$

$$a_5^2 = \frac{16}{\cancel{9}} \cdot \cancel{149}^{16}$$

$$a_5 = \pm \sqrt{256}$$

$$a_5 = +16$$

(P.G. oscilante)

$$\left(\overset{+}{a_3}, \overset{-}{a_4}, \overset{+}{a_5}, \overset{-}{a_6}, \overset{+}{a_7}\right)$$

02. **Correta.**

$$S_\infty = \frac{a_1}{1-q} = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8$$

04. **Incorreta.**

(6 + x, 10 + x, 15 + x)

$(10 + x)^2 = (6 + x) \cdot (15 + x)$

$100 + 20x + \cancel{x^2} = 90 + 6x + 15x + \cancel{x^2}$

10 = x

P.G. (16, 20, 25)

$$q = \frac{20}{16} = \frac{5}{4}$$

08. **Incorreta**. É P.G. de razão 1 ou P.A. de razão zero.

4.13) D

$S_8 = ?$

$$\begin{cases} a_4 - a_1 = 21 \\ a_3 - a_1 = 9 \end{cases}$$

$$\begin{cases} a_1q^3 - a_1 = 21 \\ a_1q^2 - a_1 = 9 \end{cases}$$

$$\begin{cases} \cancel{a_1} \cdot (q^3 - 1) = 21 \\ \cancel{a_1} \cdot (q^2 - 1) = 9 \end{cases} \quad (\div)$$

$$\frac{q^3 - 1}{q^2 - 1} = \frac{7}{3}$$

Observação: $q^3 - 1 = (q - 1) \cdot (q^2 + q + 1)$

$$\frac{\cancel{(q-1)} \cdot (q^2 + q + 1)}{\cancel{(q-1)} \cdot (q + 1)} = \frac{7}{3}$$

$3q^2 + 3q + 3 = 7q + 7$

$3q^2 - 4q - 4 = 0$

$q' = 2$

$\cancel{q'' = \frac{-2}{3}}$

$a_1 = 3$

P.G. (3, 6, 12, ...)

$$S_8 = \frac{a_1 \cdot (q^8 - 1)}{q - 1} = \frac{3 \cdot (2^8 - 1)}{2 - 1} = 765$$

4.14) A

P.G. $(x, xq, xq^2, xq^3, xq^4, \dots)$

Ordem par: $(xq, xq^3, xq^5, \dots)$ razão: q^2

$a_2, \ a_4, \ a_6, \dots$

$$10 = \frac{xq}{1 - q^2} \Rightarrow 1 - q^2 = \frac{xq}{10} \quad \text{(I)}$$

Ordem ímpar: $(x, xq^2, xq^4, \dots)$ razão: q^2

$a_1, a_2, a_3, \dots$

$$20 = \frac{x}{1 - q^2} \Rightarrow 1 - q^2 = \frac{x}{20} \quad \text{(II)}$$

Através de I e II, encontramos:

$$\frac{xq}{10} = \frac{x}{20} \Rightarrow q = \frac{1}{2}$$

Usando II, temos:

$$1 - q^2 = \frac{x}{20}$$

$$1 - \frac{1}{4} = \frac{x}{20}$$

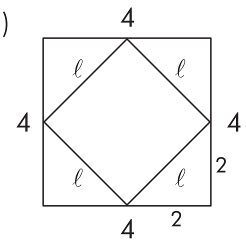
20 – 5 = x

15 = x

$$\text{P.G.} \left(15, \frac{15}{2}, \frac{15}{4}, \dots\right)$$

$$a_3 = \frac{15}{4}$$

4.15)



$\ell^2 = 2^2 + 2^2$

$\ell^2 = 8$

Áreas

$A_1 = 4^2 = 16$

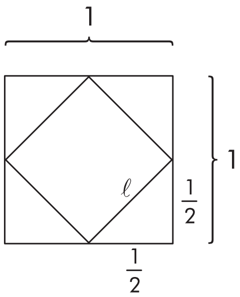
$A_2 = \ell^2 = 8$

P.G. (16, 8, ...)

$q = \frac{1}{2}$

$S_\infty = \frac{16}{1-\frac{1}{2}} = \frac{16}{\frac{1}{2}} = 32$

4.16)



$\ell^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$\ell^2 = \frac{1}{4} + \frac{1}{4}$

$\ell^2 = \frac{2}{4}$

$\ell = \frac{\sqrt{2}}{2}$

Perímetros:

$Q_1 = 4 \,.\, 1 = 4$

$Q_2 = 4 \,.\, \frac{\sqrt{2}}{2} = 2\sqrt{2}$

P.G. $(4, 2\sqrt{2}, ...)$

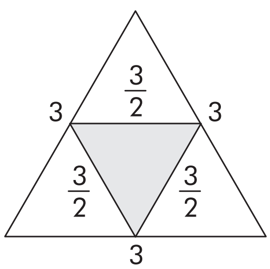
$q = \frac{\not{2}\sqrt{2}}{\not{4}} = \frac{\sqrt{2}}{2}$

$S_\infty = \frac{4}{1-\frac{\sqrt{2}}{2}} = \frac{4}{\frac{2-\sqrt{2}}{2}} = 4 \,.\, \frac{2}{2-\sqrt{2}} =$

$= \frac{8}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{8(2+\sqrt{2})}{4-2} =$

$= \frac{8(2+\sqrt{2})}{2} = 4(2+\sqrt{2})$

4.17)



Observação: Num triângulo qualquer, o segmento que une os pontos médios de dois lados é paralelo ao terceiro lado e tem medida igual à metade do terceiro lado.

a) Perímetros

P.G. $\left(9, \frac{9}{2}, ...\right)$

$S_\infty = \frac{9}{1-\frac{1}{2}} = \frac{9}{\frac{1}{2}} = 18$

b) Áreas

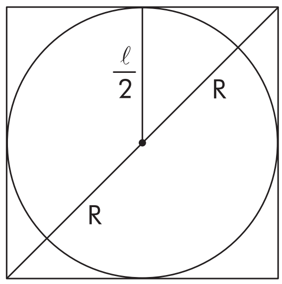
$A_1 = \frac{\ell^2\sqrt{3}}{4} = \frac{9\sqrt{3}}{4}$

$A_2 = \frac{\frac{9}{4} \,.\, \sqrt{3}}{4} = \frac{9\sqrt{3}}{16}$

P.G. $q = \frac{1}{4}$

$S_\infty = \frac{\frac{9\sqrt{3}}{4}}{1-\frac{1}{4}} = \frac{\frac{9\sqrt{3}}{4}}{\frac{3}{4}} = \frac{\not{9}\sqrt{3}}{\not{4}} \cdot \frac{\not{4}}{\not{3}} = 3\sqrt{3}$

4.18)



$R = \frac{\text{diagonal}}{2} = \frac{\ell\sqrt{2}}{2}$

$\ell = \frac{2R}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\ell = \frac{2R\sqrt{2}}{2} = R\sqrt{2}$

1º raio: R

2º raio: $\frac{\ell}{2} = \frac{R\sqrt{2}}{2}$

P.G. $\left(R, \frac{R\sqrt{2}}{2}, ...\right)$

$q = \frac{\frac{R\sqrt{2}}{2}}{R} = \frac{\sqrt{2}}{2}$

$S_\infty = \frac{R}{1-\frac{\sqrt{2}}{2}} = \frac{R}{\frac{2-\sqrt{2}}{2}}$

$$S_\infty = R \cdot \frac{2}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{2R\,.\,(2+\sqrt{2})}{4-2} =$$

$$= \frac{\cancel{2}R.(2+\sqrt{2})}{\cancel{2}} = R\,.\,(2+\sqrt{2})$$

4.19) Área 1: $1^2 = 1$

Área 2: $3 \cdot \left(\frac{1}{3}\right)^2 = 3 \cdot \frac{1}{9} = \frac{1}{3}$

Área 3: $9 \cdot \left(\frac{1}{9}\right)^2 = 9 \cdot \frac{1}{81} = \frac{1}{9}$

P.G. $\left(1, \frac{1}{3}, \frac{1}{9}, \ldots\right)$

$$S_\infty = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

4.20) B

1º: x
2º: 2x
3º: 4x
4º: 8x
5º: 16x
$x + 2x + 4x + 8x + 16x = 7440$
$31x = 7440$
$x = 240$

4.21) C

1997: 520
1998: 1040
1999: 2080
2000: 4160
2001: 8320
2002: 16640

4.22) E

P.G. (111; 123,21; ...)

$$q = \frac{123{,}21}{111} = 1{,}11$$

$a_{12} = a_1 \cdot q^{11} =$
$= 111 \cdot (1{,}11)^{11} \cong 111 \cdot 3{,}1517 \cong 349{,}8387$

4.23) a) 2,5454...

$= 2 + 0{,}54 + 0{,}0054 + 0{,}000054 + \ldots$

$= 2 + \frac{54}{100} + \frac{54}{10000} + \frac{54}{1000000} + \ldots$

$$S_\infty = \frac{\frac{54}{100}}{1-\frac{1}{100}} = \frac{\frac{54}{100}}{\frac{95}{100}} = \frac{54}{95}$$

$$2{,}5454\ldots = 2 + \frac{54}{99} = \frac{252}{\cancel{99}} = \frac{28}{11}$$

b) 2,95454 ...
$= 2 + 0{,}9 + 0{,}054 + 0{,}00054 + \ldots$

$= 2 + \frac{9}{10} + \frac{54}{1000} + \frac{54}{100000} + \ldots$

P.G. $q = \frac{1}{100}$

$$S_\infty = \frac{\frac{54}{1000}}{1-\frac{1}{100}} = \frac{54}{\cancel{1000}} \cdot \frac{\cancel{100}}{99} = \frac{54}{990}$$

$$2{,}95454\ldots = 2 + \frac{9}{10} + \frac{\cancel{54}}{\cancel{990}} =$$

$$= 2 + \frac{9}{10} + \frac{6}{110} = 2 + \frac{9}{10} + \frac{3}{55} =$$

$$= \frac{220+99+6}{110} = \frac{\cancel{325}}{\cancel{110}} = \frac{65}{22}$$

4.24) B

$$\sqrt{1{,}777\ldots} = \sqrt{1\frac{7}{9}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

$$\sqrt{0{,}111\ldots} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

$$\frac{\sqrt{1{,}777\ldots}}{\sqrt{0{,}111\ldots}} = \frac{\frac{4}{3}}{\frac{1}{3}} = 4$$

4.25) a) $E = \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}}}}} = \sqrt[32]{2}$

b) $E = \sqrt{2\sqrt{2\sqrt{2\sqrt{2\ldots}}}} = 2^{\frac{1}{2}} \cdot 2^{\frac{1}{4}} \cdot 2^{\frac{1}{8}} \cdot 2^{\frac{1}{16}} \ldots =$

$$= 2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots} = 2^{\frac{\frac{1}{2}}{1-\frac{1}{2}}} = 2^{\frac{\frac{1}{2}}{\frac{1}{2}}} = 2$$

4.26) C

(1, 1, 2, 3, 5, 8, 13, 21, ...)
Note que cada termo, a partir do terceiro, é igual à soma dos dois anteriores, isto é;

$a_{n+1} = a_n + a_{n-1}$, $n \in N^*$, $n \geq 2$